Due date: Nov 3 (in class)
Collaboration is encouraged. However the writeup should be your own. You should cite all collaborators and any other references that you might be using.

1. Show that if P = NP, then EXP = NEXP.

2. In class we showed that the class NL can (alternatively) be defined as the class of languages \( L \) that are decidable by a deterministic log-space Turing machine \( M \) (i.e. the work tape can only use logarithmic space) with one extra read-once tape (whose head only allowed to stay in place or move to the right). The read-once tape is allowed to contain a polynomial length witness, and an input \( x \in L \) if and only if there is some witness that can make \( M \) accept. Show that if the read-once tape is replaced by a tape on which the head can move both ways (but is still read-only), then such Turing machines can decide any language in NP.

3. Describe a language \( L \) that is decidable, and is contained in \( P/poly \), but is not contained in \( P \).

4. We say that a language \( L \subseteq \{0,1\}^* \) is sparse if there is some polynomial \( p : \mathbb{N} \to \mathbb{N} \) such that for all \( n \), \( |L \cap \{0,1\}^n| \leq p(n) \). Show that if a sparse language is NP-complete then the polynomial hierarchy collapses.

5. State a complete problem for each level of the polynomial hierarchy, and sketch a proof that it is complete under polynomial time reductions. Also show that if for any \( i \), \( \sum_p^p = \prod_p^p \), then the polynomial hierarchy collapses to the \( i \)th level.

6. Let ZPP be the class of languages \( L \) that can be decided by a probabilistic Turing machine \( M \) that halts on each input in expected polynomial time, and for each input \( x \), \( \Pr[M(x) = L(x)] = 1 \). Prove that \( ZPP = RP \cap \text{co-RP} \).