Errata to:

Eduardo D. Sontag

“Universal nonsingular controls”


The last paragraph of this paper consists of a remark sketching how to derive, in an alternative way, one of the main steps in the proof of a recent theorem by Coron. There is a minor but annoying mistake in this paragraph. (The main results of the paper are not affected in any way.)

In the remark in question, one starts with a smooth universal control \( \omega \) and then obtains a family of controls \( \nu(x) \), parameterized by the initial state \( x \); this family is as needed in Coron’s proof. Unfortunately, the construction of \( \nu(x) \) from \( \omega \) was stated wrong (we thank Leonid Gurvits for pointing this out). Indeed, the formula used there, namely \( \nu(x) = \rho(\|x\|^2) \omega(\cdot) \), is not assured of preserving nonsingularity, contrary to what is said in the paper; it is obvious that the time should have been rescaled as well. Thus, the correct formula is:

\[
\nu(x) = \rho(\|x\|^2) \omega(\rho(\|x\|^2)(\cdot)) .
\]

Unfortunately, this complicates matters a bit, as now the control is defined on a varying interval \([0,T/\rho(\|x\|^2)]\), and it may result in explosions even if \( \rho(\cdot) \) is small. However, assuming that the control was not just smooth but analytic –the result insures existence of analytic universal nonsingular controls– one can observe that universality still holds for the restriction of \( \nu(x) \) to any nontrivial subinterval \([0,\varepsilon]\), and for \( \varepsilon = \varepsilon(x) \) there will be no explosions, for a suitable choice of smooth \( \varepsilon(x) \). The rest of the remark follows the same outline as in the paper. At one point one needs to patch so as to make derivatives vanish at all orders, and for this one needs to observe that nonsingularity is preserved when the control is modified only on a small enough interval.