

P2C2: Rethinking Mathematics Education

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Introduction & Motivation

Socrates said that all truths are already inherent in the human mind. The Socratic Method involves the teacher guiding the pupil to knowledge by asking penetrating questions that tease out that knowledge from the pupil. Socrates demonstrates this by leading a slave boy “ignorant of geometry” to discover the Pythagorean Theorem in the special case of an isosceles right triangle!

Socrates begins by making the pupil realize his initial idea, that one can double the area of a square by doubling its side, is wrong, and that one-and-a-half times the side won't do it either, causing the boy to become puzzled. That state of bewilderment (*áporía*), according to Socrates, is a key step in education: *“whereas [the pupil] thought then that he did know it, and answered confidently like someone who knows, and did not think himself in any difficulty, now he actually thinks he is in trouble, he doesn't know and likewise does not think he knows.* [1]”

The Socratic, or **Active Learning Method**, was introduced in the Math Department more than 20 years ago via *workshops* for Calculus courses. In workshops students work on problems together in groups, with the workshop instructor facilitating their discussion - not solving the problems for them, but leading them to the solution through an interactive process.

This method is one of the most effective ways of increasing student performance in STEM [2], and a characteristic of successful Calculus programs [3]. According to the MAA [3], another characteristic of successful programs is **continuous experimentation**. In this vein, the Math Department is undertaking a major overhaul of all Calculus instruction. The goal is to enhance areas of strength and bring about change in key areas, including the effectiveness of placement procedures.

Methods

1. RUPReady Tests: In accordance with new university policies, the placement exams which determine a student's entry level in the math program are now administered online. Such exams are crucial, but far from perfect, and their reliability has decreased markedly since moving online. The current strategy is to use the placement exam to determine the “highest possible placement” of each student, and to caution students about their responsibility to assess their own ability and to consider registering for a lower-level course. We are taking steps to improve placement, including (1) statistical studies to determine which preliminary skills are most crucial, (2) improved advising to counsel students with marginal skills to drop down from Calculus to PreCalculus, and (3) administering an additional diagnostic exam (the RUPReady test) on the first day of class.

2. Improvements in Active Learning: While the Math Dept. has been a pioneer in Active Learning techniques at Rutgers, there are many pedagogical and technological innovations that need to be incorporated into our Calculus curriculum in order to bring us up-to-date. Some of these are:

- (a) Piloting clicker-style questions to keep students engaged during lectures.
- (b) Creating new workshop problems that are complex yet approachable, which help students learn new topics more deeply than by simply listening or attempting straight-forward problems.
- (c) Increasing small group discussions among students to correct misconceptions early.
- (d) Experimenting with the flipped classroom format (fully flipped, semi-flipped, flipped hybrid, etc.)

3. Learning Assistants:

This innovative program at Rutgers engages undergraduates in classroom teaching and learning while helping them develop their teaching, leadership, and interpersonal skills. Learning Assistants (LAs) assist in the classroom in several ways, including facilitating discussions and workshops during lectures, team-teaching workshop sessions with a graduate teaching assistant, or independently leading supplemental study groups or recitations. LA's are an essential component of the math department's overhaul of calculus workshops, since Active learning requires interaction on an individual, or at least a small group, basis, and students are known to benefit from interaction with mentors who, being undergraduates, are their own peers. There will be one LA per workshop in Calculus I & II, and two LA's per class for the new Math 123 course.

4. Three-Tiered Test Banks:

To facilitate the construction of exams that test students' mastery of the material at an appropriate and consistent level, we have set up, and are growing, banks of test questions at three levels. The levels correspond to mastery at the C, B, and A grade levels. Students earning C's and B's should generally be able to get partial credit on A-level problems. A test is then 60% C problems, 20% B problems and 20% A problems. A similar list is available to students for practice. Later, each problem will be classified according to the particular learning goals of the course that it is aimed at assessing, so that students can better use their practice exams to guide their studies.

5. Drop-down & Prep-up:

Students who fail the RUPReady Test three times during the first 2.5 weeks of the semester are counseled to drop down to the lower course, i.e. from Calculus I to Intensive PreCalculus, or from PreCalculus to Intermediate Algebra. In spite of this, we expect to continue to have a significant number of students who opt to take Calculus with inadequate preparation. These students very often end up withdrawing from the course or failing. Not only do they lose the semester, but they are no better prepared to take Calculus the next semester.

To tackle this problem, we are initiating a new course “640:123 Preparation for Calculus”. This is a 2 credit course that runs the final 8 weeks of the semester and will be available to students as a “drop down” option from Calculus. The course will not be a fast replay of PreCalculus; it will be specially tailored to systematically target specific common weaknesses in background (from elementary algebra through PreCalculus) that block success in Calculus.

From the beginning, these sections will be designed with active learning and “flipped classroom” techniques; Students will make use of online video materials to review routine techniques, and classroom time will focus on addressing student difficulties. Since students in this course, who have already stumbled on Calculus, are likely to require additional attention, the course will be planned to be taught in classes of 30 (rather than the 75-90 that is typical for PreCalculus). Students who switch from Calculus I to Math 123 will not receive a W for the Calculus I course.

6. Learning Center's Study Groups:

The RUPReady Tests also help us identify a certain segment of population who are specially at-risk of failing our entry-level courses, so that we can set them up with an early intervention program. These are students who either have a marginally passing score in the RURT, or are otherwise ineligible for dropping down to a lower course because they already have the lower course on their transcript. This group of students will have the opportunity of signing up for a special Study Groups program offered by the Learning Center that has both math as well as study skills content, while also emphasizing the importance of proper sleep, nutrition and exercise. The LA program will be collaborating with the Academic Coaching program on this and the LAs will work side by side with an academic coach who has particular experience in math. The study groups are currently being set up. There will be 2 LAs for PreCalculus and 5 for Calculus.



RUTGERS

CAN Calculus BE TAUGHT?

Yes, but not before students are convinced that they do not know it already. (Just ask Socrates!)

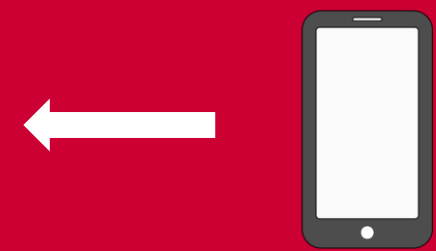
The key to improving outcomes and learning experiences in Calculus courses is a multi-pronged approach that begins with making sure the initial conditions are right.

MATH DEPARTMENT'S P2C2 EFFORTS:

1. Accurate placement.....RUPReady Tests
2. Socratic method.....Active Learning
3. Student mentors.....Learning Assistants
4. Uniform standards.....3-Tiered Test Banks
5. Multiple safety nets.....Drop-down & Prep Up
6. Extended support system.....LC Study Groups



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Supporting Data

- **Importance of placement procedures and safety nets:** “Though [accurate placement] could be considered part of the first characteristic of successful programs, it received so much attention from all of the universities that we have elevated it to the level of a separate point. These universities evaluate and adjust their placement procedures on an annual basis. We also found a great deal of attention paid to those students near the cutoff, paying particular attention to programs in support of those allowed into Calculus I but most at risk and working with those who did not quite make the cut so that they were placed in programs that addressed their actual needs.” [3]
- **The MAA on Active Learning:** “Classroom practices aimed at fostering student engagement attend to the research-based idea that students learn best when they are engaged in their learning (e.g., Freeman, et al., 2014). Consistent use of active learning strategies in the classroom also provide a pathway for more equitable learning outcomes for students with demographic characteristics who have been historically underrepresented in science, technology, engineering, and mathematics (STEM) fields (e.g., Laursen, Hassi, Kogan, and Weston, 2014).

- **On flipped classrooms:** “There was a significant increase in students' self-efficacy over the course of the term in flipped Math 152 [Calculus II]” (Kerrigan et al, 2019):

N=23	Calculus Self-Efficacy	
	Pre-Survey	Post-Survey
Mean	5.30	7.04
St. Dev	2.38	1.33

Matched pairs two-tailed t-test:
p=0.0001*, t=4.729

* All students either increased or stayed the same from pre-survey to post-survey on their self-ranking of Calculus Self-Efficacy



Pictures from Prof. Kerrigan's flipped Math 152 (Calculus II), summer 2019

- **On the effectiveness of active learning:** “To test the hypothesis that lecturing maximizes learning and course performance, we meta-analyzed 225 studies that reported data on examination scores or failure rates when comparing student performance in undergraduate science, technology, engineering, and mathematics (STEM) courses under traditional lecturing versus active learning. The effect sizes indicate that on average, student performance on examinations and concept inventories increased by 0.47 SDs under active learning ($n = 158$ studies).”

“These results indicate that average examination scores improved by about 6% in active learning sections.”

“Students in classes with traditional lecturing were 1.5 times more likely to fail than were students in classes with active learning.”

“If the experiments analyzed here had been conducted as randomized controlled trials of medical interventions, they may have been stopped for benefit—meaning that enrolling patients in the control condition might be discontinued because the treatment being tested was clearly more beneficial.” [2]

- **Sample comments from student evaluations of Prof. Kerrigan's flipped classroom:**

What do you like best about this course?

These comments are intended for all instructors.

Comments
I liked the format of the course rather than a standard lecture.
I used to dislike mathematics but Prof Kerrigan really made me love it again. I like the challenging aspects of the course, like the law of cosines.
I liked the active learning class and I thought John was a great teacher.
The teacher by far kerrigan is the best teacher i have ever taken in rutgers so far
How the class was very involved. The grading and % were fair and homework was very helpful
I did not have a strong love for the material of this course. However, I enjoyed the active learning classroom environment.
The flipped learning style is unique and allows students to work together in class.
The “Flipped” classroom learning environment was one of the best, if not the best college level classroom experiences I've had. I had discovered over the last semesters that I learn best when I work through problems at my own pace and can ask questions as they appear. What that usually meant is, aside from the hours in class, I would have to spend a large amount of time outside of class for supplemental instruction.
This style of teaching made the logical step of having the person most knowledgeable of the subject matter (the professor) available for the students the entire class period. My questions and/or misunderstandings were resolved quickly, which allowed for me to constantly be progressing forward. I never felt like I was behind or didn't understand a section.
Also, the seating arrangement was something I took for granted at first, but in retrospect is very useful. I could always turn to my peers for quick questions or reaffirmation of concepts. I also really enjoyed answering questions, as I find that when I can clearly and efficiently explain to someone else, then the concept gets cemented in my own head.

Future Directions

- **Resources and training for instructors:** Few instructors have experience with active learning methods. Effective use of such methods on a large scale will require preparation of lecture-by-lecture guides to lectures, and systematic training of instructors in these methods.
- **Developing Versions of Basic Courses for specific audiences:** At the request of the School of engineering, we are developing a version of precalculus 115 for Engineering students. We have preliminary discussions with Economics concerning a two semester sequence of calculus and other mathematics for Economics majors (who currently take 135-136 which combines Life Science, Economics and Business majors.) We will invite the Business School to these discussions, since their needs are likely to be similar to Economics.
- **Development and/or curating of online videos to cover routine technical material:** One of the key difficulties in introducing active learning methods is that the time devoted to having students grapple with problems, and troubleshooting their difficulties takes away from time to present these basic techniques. An important aspect for freeing up time is to move presentation of routine techniques to an online format that is accessible to students outside of lecture. We will create an organized repository of such materials from a combination of existing sources, and new material.

References & Acknowledgments

1. Plato's *Meno*, translated by Cathal Woods, Available at <http://dx.doi.org/10.2139/ssrn.1910945>
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3. Bressoud & Rassmussen, “Seven characteristics of successful Calculus programs,” Notices of AMS, **62**:2, 144-146 (2015) https://www.maa.org/sites/default/files/pdf/cspcc/seven_characteristics.pdf
4. Kerrigan et al, “Self-efficacy in a flipped Calculus II classroom,” Rutgers University Active Learning Symposium (2019) <http://bit.ly/2WbAoFA>

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