We prove that if $X$ is a locally finite $n$-dimensional polyhedron such that $X \times I^\infty$ admits a $\mathcal{Z}$-compactification, then $X \times I^{2n+5}$ also admits a $\mathcal{Z}$-compactification. Our argument makes use of an extension of Dierker’s lemma which, in its original form, says that if a finite polyhedron $X$ collapses to a finite polyhedron $Y$, then there is a $q$ such that $Y \times I^q$ collapses to $X$. 