1. Remove the lower left corner square and the upper right corner square from an ordinary 8-by-8 chessboard. Can the resulting board be covered by 31 dominos? Assume each domino will cover exactly two adjacent squares of the board.

2. Let thirteen points be given in the plane and suppose they are connected by the segments $P_1P_2, P_2P_3, \ldots, P_{12}P_{13}, P_{13}P_1$. Is it possible to draw a straight line which passes through the interior of each of these segments?

3. Is it possible to trace a path along the arcs of the figure below which traverses each arc once and only once? (Hint: Count the number of arcs coming out of each vertex.)

4. Is it possible to trace a path along the figure below which passes through each juncture point once and only once? (Hint: Color the vertices in an alternating manner.)

5. Prove that there are infinitely many integers $n$ so that $n$, $n + 1$, $n + 2$ are each the sum of two squares of integers. (Example: $0 = 0^2 + 0^2$, $1 = 0^2 + 1^2$, $2 = 1^2 + 1^2$.)

6. Let $a_j$, $b_j$, $c_j$ be integers for $1 \leq j \leq N$. Assume, for each $j$, at least one of $a_j$, $b_j$, $c_j$ is odd. Show that there exist integers $r$, $s$, $t$ such that $ra_j + sb_j + tc_j$ is odd for at least $4N/7$ values of $j$, $1 \leq j \leq N$. 

Easy Putnam Problems

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Easy Putnam Problems
7. Right triangle $ABC$ has right angle at $C$ and $\angle BAC = \theta$; the point $D$ is chosen on $AB$ so that $|AC| = |AD| = 1$; the point $E$ is chosen on $BC$ so that $\angle CDE = \theta$. The perpendicular to $BC$ at $E$ meets $AB$ at $F$. Evaluate $\lim_{\theta \to 0} |EF|$. (Here $|PQ|$ denotes the length of the line segment $PQ$.)

![Diagram of right triangle ABC with additional points D and E and a perpendicular from E to AB at F.]

8. Find necessary and sufficient conditions on positive integers $m$ and $n$ so that

$$
\sum_{i=0}^{mn-1} (-1)^{[i/m]+[i/n]} = 0.
$$

9. Let $N$ be the positive integer with 1998 decimal digits, all of them 1; that is

$$
N = 1111 \cdots 11.
$$

Find the thousandth digit after the decimal point of $\sqrt{N}$.

10. Let $p(x)$ be a polynomial that is nonnegative for all real $x$. Prove that for some $k$ there are polynomials $f_1(x), f_2(x), \ldots, f_k(x)$ so that

$$
p(x) = \sum_{j=1}^{k} (f_j(x))^2.
$$