1. (11 pts) Find the equation of the plane which passes through the points (1,2,3), (3,-2,1), and (-1,2,1). (Ans: \( x + y - z = 0 \).)

2. (12 pts) Find the curvature of the ellipse \( x = 3 \cos t, \ y = 4 \sin t \) at the points (3,0) and (0,4). (Ans: \( \kappa = 4 \) and \( \kappa = 3 \).)

3. (11 points) Find the center and the radius of the sphere \( x^2 + y^2 + z^2 + 4x + 6y - 10z + 2 = 0 \). (Ans: \( (x + 2)^2 + (y + 3)^2 + (z - 5)^2 = 36 \), so center is \((-2,-3,5)\) and radius is 6.)

4. (12 points) If \( xyz^2 + x^3 y^2 z = x + y + z \), find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \).
(Ans: \( \frac{\partial z}{\partial x} = -\frac{y^2 z^3 + 3x^2 y^2 z - 1}{3x y^2 z^2 + x^3 y^2 - 1}, \ \frac{\partial z}{\partial y} = -\frac{2x y z^3 + 2x^3 y z - 1}{3x y^2 z^2 + x^3 y^2 - 1} \))

5. (12 points) Find the equations of the tangent plane and normal line to the surface given by \( x^2 y + x z^2 + y^2 z = -1 \) at the point (1,2,-1). (Ans: \( 5(x - 1) - 3(y - 2) + 2(z + 1) = 0, \ \mathbf{r}(t) = \langle 1, 2, -1 \rangle + t\langle 5, -3, 2 \rangle \).

6. (12 points) Write the integral \( \int_{-1}^{1} \int_{x^2}^{1} f(x, y) \, dy \, dx \) as an integral \( dx \, dy \).
(Ans: \( \int_{0}^{1} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y) \, dx \, dy \).)

7. (12 points) Find the area enclosed by one loop of the 4-leafed rose \( r = \cos 2\theta \). (Ans: \( \frac{\pi}{8} \).)

8. (12 points) Find the volume that lies under the paraboloid \( z = x^2 + y^2 \) and over the triangle with vertices (1,0,0), (0,1,0), and (0,0,0). (Ans: \( \frac{1}{6} \).)

9. (11 points) Find the volume of the solid enclosed by the cylinder \( x = y^2 \) and the planes \( z = 0 \) and \( x + z = 1 \). (Ans: \( \frac{8}{15} \).)

10. (12 points) Convert the integral \( \int_{-1}^{1} \int_{\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} f(x, y, z) \, dz \, dy \, dx \) to an equivalent integral in cylindrical coordinates.
(Ans: \( \int_{0}^{2\pi} \int_{y^2}^{2-y^2} f(r \cos(\theta), r \sin(\theta), z) \, r \, dz \, dr \, d\theta \).)

11. (12 points) Convert the integral \( \int_{-1}^{1} \int_{\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} f(x, y, z) \, dz \, dy \, dx \) to an equivalent integral in cylindrical coordinates.
(Ans: \( \int_{0}^{2\pi} \int_{y^2}^{2-y^2} f(r \cos(\theta), r \sin(\theta), z) \, r \, dz \, dr \, d\theta \).)

12. (12 points) Convert the integral \( \int_{-1}^{1} \int_{\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} f(x, y, z) \, dz \, dy \, dx \) to an equivalent integral in cylindrical coordinates.
(Ans: \( \int_{0}^{2\pi} \int_{y^2}^{2-y^2} f(r \cos(\theta), r \sin(\theta), z) \, r \, dz \, dr \, d\theta \).)
11. (11 points) Evaluate \( \int_C y \sin z \, ds \) where \( C \) is the helix given by \( x = \cos t, \ y = \sin t, \ z = t, \ 0 \leq t \leq 2\pi \). (Ans: \( \sqrt{2\pi} \).)

12. (12 points) Find the local maxima, local minima, and saddles of \( f(x, y) = x^3 + 3xy - y^3 \). (Ans: Saddle at \((0,0)\). Local min at \((1,-1)\).)

13. (12 points) If \( u = x^y, \ x = \sin t, \ y = \cos t \), find \( \frac{du}{dt} \) when \( t = \frac{\pi}{4} \).
Ans: \( \sin(t)\cos(t) \left( -\sin(t) \ln(\sin(t)) + \frac{\cos(t)^2}{\sin(t)} \right) \)

14. (12 points) Let \( \mathbf{F} = 2xy^3z^4 \mathbf{i} + 3x^2y^2z^4 \mathbf{j} + 4x^2y^3z^3 \mathbf{k} \). Find \( f \) with \( \nabla f = \mathbf{F} \) and use it to evaluate the integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( \mathbf{r} = t \mathbf{i} + t^2 \mathbf{j} + t^3 \mathbf{k}, \ 0 \leq t \leq 2 \). (Ans: 1048576.)

15. (12 points) Use Green’s Theorem to find the work done by \( \mathbf{F} = x(x+y) \mathbf{i} + 2xy^2 \mathbf{j} \) in moving a particle from the origin along the x-axis to \((1,0)\), then along the line segment from \((1,0)\) to \((0,1)\), and then back to the origin along the y-axis. (Ans: 0.) For no particular reason I can see - it just comes out that way.

16. (12 points) Use the Divergence Theorem to calculate the integral \( \int \int_S \mathbf{F} \cdot d\mathbf{S} \) where \( \mathbf{F} = xy^2 \mathbf{i} + yz \mathbf{j} + zx^2 \mathbf{k} \) and \( S \) is the surface of the solid that lies between the cylinders \( x^2 + y^2 = 1 \) and \( x^2 + y^2 = 4 \) and between the planes \( z = 1 \) and \( z = 3 \). (Ans: 27\pi.)

17. (12 points) Evaluate the surface integral \( \int \int_S \mathbf{F} \cdot d\mathbf{S} \) where \( \mathbf{F} = e^y \mathbf{i} + ye^x \mathbf{j} + x^2y \mathbf{k} \) and \( S \) is the part of the paraboloid \( z = x^2 + y^2 \) that lies above the square \( 0 \leq x \leq 1, \ 0 \leq y \leq 1 \), and has upward orientation. (Ans: \( -\frac{5}{3}e + \frac{11}{6} \).)

**Bonus Problem:** Find the volume of the intersection of the two cylinders \( x^2 + y^2 = 1 \) and \( x^2 + z^2 = 1 \). Can you find the volume of the triple intersection of these two and \( y^2 + z^2 = 1 \)? (Ans: The first part was on midterm #2. The second part is a pain.)