1. Find the area of the parallelogram with vertices 
\(A(1, 1, 1), B(2, -1, 4), C(3, 5, -4), D(4, 3, -1)\).

Hint: Two of the sides of the parallelogram are \(AB\) and \(AC\).

\(AB = \langle 1, -2, 3 \rangle\) and \(AC = \langle 2, 4, -5 \rangle\). \(AB \times AC = \langle -2, 11, 8 \rangle\) and \(|AB \times AC| = \sqrt{189}\).

2. Find the equation of the plane through \((6, 0, -2)\) which contains the line 
\(r(t) = \langle 4, 1, 6 \rangle + t\langle -3, 4, 1 \rangle\).

Letting \(t = 0\) and \(t = 1\), we see that \(A(4, 1, 6), B(1, 5, 7),\) and \(C(6, 0, -2)\) lie in the plane. 
\(AB = \langle -3, 4, 1 \rangle\) and \(AC = \langle 2, -1, -8 \rangle\), so \(AB \times AC = \langle 31, 22, 5 \rangle\). The equation of the plane is 
\(31(x - 6) + 22(y - 0) + 5(z + 2) = 0\).

3. Find parametric equations for the tangent line to the curve 
\(r(t) = \langle \cos t, 3e^{2t}, 3e^{-2t} \rangle\) at the point \((1, 3, 3)\).

\(r'(t) = \langle \cos(t), 3e^{2t}, 3e^{-2t} \rangle\)

\(r(0) = (1, 3, 3)\) and \(r'(0) = (0, 6, -6)\), so the vector equation of the tangent line is 
\(r(t) = \langle 1, 3, 3 \rangle + t\langle 0, 6, -6 \rangle\)

and the parametric equations of the tangent line are \(x = 1, y = 3 + 6t, z = 3 - 6t\).

4. Find the curvature of the parabola \(x = t, y = 2t^2 + 1\) at the point \((2, 9)\).

\(r(t) = \langle t, 2t^2 + 1, 0 \rangle\)

\(r'(t) = \langle 1, 4t, 0 \rangle\)

\(r''(t) = \langle 0, 4, 0 \rangle\)

\[\kappa = \frac{|r'(2) \times r''(2)|}{|r'(2)|^3} = \frac{4}{65^{3/2}}\]

5. If \(f(x, y) = \frac{x^2y^2}{(x^4 + y^4)},\) does \(\lim_{(x,y)\to(0,0)} f(x, y)\) exist? Explain your answer.

If \(x = 0, y = t, \frac{\partial^2 f}{\partial t^2} = 0\), so \(\lim_{t \to 0} f(0, t) = 0\).

If \(x = t, y = t, \frac{\partial^2 f}{\partial t^2} = \frac{1}{2}\), so \(\lim_{t \to 0} f(t, t) = 1/2\).

Since these are different, the limit does not exist.
6a. If \( z = xe^{\frac{y}{x}} \), find \( dz \).

\[
dz = \frac{\partial z}{\partial x} \, dx + \frac{\partial z}{\partial y} \, dy = \left( e^{\frac{y}{x}} - \frac{y}{x} e^{\frac{y}{x}} \right) dx + e^{\frac{y}{x}} dy
\]

6b. If \( z = g(ax^2y, by^2) \), find \( \frac{\partial z}{\partial y} \).

Set \( u = ax^2y, v = by^2 \).

\[
\frac{\partial z}{\partial y} = \frac{\partial g}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial g}{\partial v} \frac{\partial v}{\partial y} = \frac{\partial g}{\partial u} (ax^2) + \frac{\partial g}{\partial v} (2by)
\]

7. Find the directional derivative of the function \( f(x, y) = x^3y - 2x^2y^2 \) at the point \((1, 2)\) in the direction of \((3, 4)\). In what direction does \( f \) increase most rapidly? What is the rate of increase in that direction?

\[
\nabla f = \langle 3x^2y - 4xy^2, x^3 - 4x^2y \rangle.
\]

At the point \((1, 2)\), this is \((-10, -7)\). \( \mathbf{u} = \langle 3/5, 4/5 \rangle \) and \( D_uf = (-10, -7) \cdot \langle 3/5, 4/5 \rangle = -\frac{58}{5} \). The direction of most rapid increase is the direction of \((-10, -7)\) and the rate increase in that direction is \( \sqrt{149} \).

8. Find the linearization of the function \( f(x, y) = \frac{x}{y} \) at the point \((100, 300)\) and use it to estimate the value of 101/301.

\[
\begin{align*}
L(x, y) &= f(100, 300) + f_x(100, 300)(x - 100) + f_y(100, 300)(y - 300) \\
\frac{f_x}{y} &= \frac{x}{y^2}, \text{ so } f_x(100, 300) = -\frac{100}{900^2} = \frac{1}{900} \\
\frac{f_y}{x^2} &= \frac{1}{y}, \text{ so } f_y(100, 300) = \frac{1}{300} \\
L(x, y) &= \frac{1}{3} + \frac{1}{300}(x - 100) - \frac{1}{900}(y - 300) \\
L(101, 301) &= \frac{1}{3} + \frac{1}{300}(1) - \frac{1}{900}(1) = \frac{151}{450}
\end{align*}
\]