1. (12 points) If \( \mathbf{v} = \langle 1, 2, 3 \rangle \) and \( \mathbf{w} = \langle -1, 1, 2 \rangle \)
   a. Find \( \mathbf{v} \cdot \mathbf{w} \). (Ans. 7)
   b. Find \( \mathbf{v} \times \mathbf{w} \). (Ans. \( \langle 1, -5, 3 \rangle \)).

2. (14 points) Find the equation of the plane through \((6, 0, -2)\) which contains the line
   \( \mathbf{r}(t) = \langle 4, 3, 7 \rangle + t \langle -2, 5, 4 \rangle \).
   (Ans. \( 33(x - 6) + 10y + 4(z + 2) = 0 \)).

3. (16 points) Find the tangential and normal components of the acceleration vector if
   \( \mathbf{r}(t) = \langle 1 + t, t^2 - 2t \rangle \).
   You get a two point bonus if your (correct) answers are vectors. (Ans. \( a_T = \frac{4t - 4}{1 + (2t - 2)^2} \langle 1, 2t - 2 \rangle \), \( a_N = \frac{-2}{1 + (2t - 2)^2} \langle 2t - 2, -1 \rangle \)).

4. (14 points) If \( e^{xz} - e^{zx^2} = 1 \), find \( \frac{\partial z}{\partial x} \). (Ans. \( \frac{-2e^{xz^2 - x^2e^{zx^2}}}{xe^{xz^2 - x^2e^{zx^2}}} \)).

5. (16 points) Find the equations of the tangent plane and normal line to the surface
   \( z = \ln(x + 2y) \) at the point \((3, -1, 0)\). (Ans. -(x - 3) - 2(y + 1) + z = 0, \( \mathbf{r}(t) = \langle 3, -1, 0 \rangle + t(-1, -2, 1) \)).

6. (14 points) Find the local maxima, minima, and saddle points for \( f(x, y) = x^2 + y^2 + x^2y + 4 \).
   (Ans. Local min \((0,0)\), saddles \((\pm \sqrt{2}, -1)\)).

7. (14 points) Change the order of integration in \( \int_1^2 \int_0^{\ln x} f(x, y) \, dy \, dx \).
   (Ans. \( \int_0^{\ln 2} \int_0^{e^y} f(x, y) \, dx \, dy \)).

8. (14 points) Find the volume of the solid that lies under the paraboloid \( z = x^2 + y^2 \), above the \( xy \)-plane, and inside the cylinder \( x^2 + y^2 = 2x \).
   (Ans. \( \frac{3\pi}{2} \)).

9. (16 points) If \( \mathbf{F}(x, y) = \langle 3 + 2xy, x^2 - 3y^2 \rangle \), find a function \( f \) such that \( \mathbf{F} = \nabla f \).
   Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( C \) is given by \( \mathbf{r}(t) = e^t \sin(t) \mathbf{i} + e^t \cos(t) \mathbf{j}, \quad 0 \leq t \leq \pi \).
   (Ans. \( e^{3\pi} + 1 \)).
10. (14 points) Set up a triple integral to find the volume of the solid bounded by the cylinder \( x = y^2 \) and the planes \( z = 0 \) and \( x + z = 1 \). YOU DO NOT NEED TO EVALUATE THIS INTEGRAL!! (Ans. \( \int_{-1}^{1} \int_{y^2}^{1-x} dz \, dx \, dy \) or \( \int_{-\sqrt{\pi}}^{\sqrt{\pi}} \int_{0}^{1-x} dz \, dx \, dy \).

11. (14 points) Find \( \int \int_{S} \mathbf{F} \cdot d\mathbf{S} \), where \( \mathbf{F} = x^3 \mathbf{i} + y^3 \mathbf{j} + z^3 \mathbf{k} \) and \( S \) is the sphere \( x^2 + y^2 + z^2 = 9 \). (Ans. \( \frac{4\pi \cdot 3^6}{5} \). Use the Divergence Theorem.)

12. (14 points) Evaluate \( \int_{C} (3y - e^{\sin x}) \, dx + (7x + \sqrt{y^4 + 1}) \, dy \), where \( C \) is the boundary of the rectangle \( 0 \leq x \leq 2, \ 0 \leq y \leq 3 \). (Ans. 24.)

13. (14 points) Find \( \int \int_{S} z \, dS \), where \( S \) is the surface with parametric equations \( x = \cos(u), \ y = \sin(u), \ z = v, \ 0 \leq u \leq 2\pi, \ 0 \leq v \leq 2 \). (Ans. \( 4\pi \).)

14. (14 points) Use Stokes’ Theorem to evaluate \( \int_{C} \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x, y, z) = \langle 2z, 4x, 5y \rangle \) and \( C \) is the curve of intersection of \( z = x + 4 \) and \( x^2 + y^2 = 4 \). \( C \) is oriented clockwise as viewed from above. (Ans. \( 4\pi \). There’s an extra “-” sign coming from the clockwise orientation.)
**Bonus Problem (5 points)** If $f$ and $g$ are twice differentiable functions, show that

$$\nabla^2(fg) = f\nabla^2 g + g\nabla^2 f + 2\nabla f \cdot \nabla g.$$