1. a. (7 pts) Find the equation of the plane which contains the points \( A(1, 1, 1) \), \( B(2, -1, 3) \) and \( C(-1, 2, 1) \).
   
b. (4 pts) What is the area of \( \triangle ABC \)?

\[
\mathbf{AB} = \langle 1, -2, 2 \rangle, \quad \mathbf{AC} = \langle -2, 1, 0 \rangle, \quad \mathbf{AB} \times \mathbf{AC} = \langle -2, -4, -3 \rangle,
\]
so the equation of the plane is
\[
-2(x - 1) - 4(y - 1) - 3(z - 1) = 0.
\]
The area of the triangle is \( \frac{\sqrt{29}}{2} \).

2. (11 points) Find a vector equation for the line through \( (1, 3, -1) \) which is parallel to the line of intersection of the planes \( x + y + z = 1 \) and \( 2x - y + 3z = 6 \).

The line is parallel to the cross product of the two normal vectors. \( \langle 1, 1, 1 \rangle \times \langle 2, -1, 3 \rangle = \langle 4, -1, -3 \rangle \), so the equation is \( \mathbf{r}(t) = (1, 3, -1) + t(4, -1, -3) \).

3. (11 points) Find the arc length of the curve \( \mathbf{r}(t) = (\cos t + t \sin t) \mathbf{i} + (t \cos t - \sin t) \mathbf{j} + t^2 \mathbf{k} \) from \( t = 0 \) to \( t = \pi/2 \).

\[
\mathbf{r}'(t) = \langle t \cos t, -t \sin t, 2t \rangle, \quad |\mathbf{r}'(t)| = \sqrt{5t^2} \quad \text{and} \quad L = \int_0^{\pi/2} \sqrt{5t} \, dt = \frac{\pi \sqrt{5}}{8}.
\]

4. (11 points) Find the tangential and normal components of the acceleration vector if \( \mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + t^3 \mathbf{k} \).

\[
a_T = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|} = \frac{4t + 18t^3}{\sqrt{1 + 4t^2 + 9t^4}}.
\]

\[
a_N = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|} = \frac{\sqrt{36t^4 + 36t^2 + 4}}{\sqrt{1 + 4t^2 + 9t^4}}.
\]

5. a. (7 points) Find \( \frac{\partial z}{\partial x} \) if \( e^{x^2+y^3+z^5} = xyz \).

b. (4 points) Find \( \frac{\partial z}{\partial x} \) if \( z = (\sin y)^{\cos x} \).

a. \( F(x, y, z) = e^{x^2+y^3+z^5} - xyz, \) so \( \frac{\partial z}{\partial x} = -\frac{F_z}{F} = -\frac{2xe^{x^2+y^3+z^5} - yz}{5ze^{x^2+y^3+z^5} - xy} \).

b. \( \frac{\partial z}{\partial x} = (\sin y)^{\cos x} \ln(\sin y)(-\sin x) \).
6. (11 points) Find \( \lim_{(x,y) \to (0,0)} \frac{(x+y)^2}{x^2+y^2} \) or show that the limit does not exist.

If \( x = 0, y = t \), \( \lim_{t \to 0} \frac{t^2}{t^2} = 1. \)

If \( x = t, y = t \), \( \lim_{t \to 0} \frac{(t+t)^2}{t^2+t^2} = 2. \)

Therefore, the limit does not exist.

7. (11 points) The pressure, volume, and temperature of a mole of an ideal gas are related by the equation \( PV = 8.31T \), where \( P \) is measured in kilopascals, \( V \) in liters, and \( T \) in kelvins. Use differentials to find the approximate change in pressure if the volume increases from 12 L to 12.3 L and the temperature decreases from 310 K to 305 K.

\[ P = \frac{8.31 T}{V}, \text{ so } dP = 8.31 \frac{dT}{V} - 8.31 \frac{T}{V^2} \quad \text{and} \quad \Delta P \approx 8.31 \left( -\frac{5}{12} \right) - \frac{8.31 \cdot 310}{12^2} \cdot .3 = -8.83 \]

8. (11 points) Find the equations of the tangent plane and normal line to the surface described by the function \( f(x, y, z) = \frac{x}{y} + \frac{y}{z} \) at the point \( (4, 2, 1) \).

\[ \nabla f = \left\langle \frac{1}{y} - \frac{x}{y^2}, -\frac{1}{z} \right\rangle = \langle 1/2, 0, -2 \rangle, \text{ so the equation of the tangent plane is} \]
\[ \frac{1}{2}(x - 4) - 2(z - 1) = 0. \text{ The equation of the normal line is } r(t) = (4, 2, 1) + t(1/2, 0, -2). \]

9. (12 points) If \( z = f(x, y) \), where \( x = r \cos \theta \) and \( y = r \sin \theta \), find \( \frac{\partial z}{\partial r} \) and \( \frac{\partial z}{\partial \theta} \) and show that
\[
\left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2 = \left( \frac{\partial z}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial z}{\partial \theta} \right)^2
\]

\[
\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial r} = \frac{\partial z}{\partial x} \cos \theta + \frac{\partial z}{\partial y} \sin \theta
\]
\[
\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial \theta} = \frac{\partial z}{\partial x} (-r \sin \theta) + \frac{\partial z}{\partial y} (r \cos \theta)
\]
\[
\left( \frac{\partial z}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial z}{\partial \theta} \right)^2 = \left( \frac{\partial z}{\partial x} \cos \theta + \frac{\partial z}{\partial y} \sin \theta \right)^2 + \frac{1}{r^2} \left( \frac{\partial z}{\partial x} (-r \sin \theta) + \frac{\partial z}{\partial y} (r \cos \theta) \right)^2
\]
\[
= \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2
\]