NOTE: These are only practice problems!

The exam will cover all the material through section 3.5.
1. Find the equation of the line that passes through (2, 4) and is perpendicular to the line 2x + 3y = 12.

2. Let \( f(x) = \sqrt{x^2 + 2x - 15}, \quad g(x) = \frac{1}{x} \)
a) Find the domain of the function \( f(x) \).
b) Find \( g(f(x)) \) and \( f(g(x)) \).
c) Find the domains of \( g(f(x)) \).

3. Find the exact value of each of the following limits. Show all work and/or give reasons for your answers:
a) \( \lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 4} \)
b) \( \lim_{x \to 4} \frac{x^2 - 5x + 6}{x^2 - 4} \)
c) \( \lim_{x \to 0} \frac{\tan 2x}{\tan 7x} \)
d) \( \lim_{x \to 0} \frac{\sqrt{3 + x} - \sqrt{3}}{x} \)
e) \( \lim_{x \to 3} \frac{|x - 3|}{x - 3} \)
f) \( \lim_{x \to 0} \frac{1 - \cos x}{x^2} \)

4. Let \( g(x) = x^5 + x^3 + 30 \). Without graphing the function \( g \), use a theorem to show that there is at least one number \( c \in (-2, 2) \) such that \( g(c) = 0 \). HINT: don’t try to find \( c \)!

5. Let \( f(x) = \begin{cases} 
  x^2 + 1 & x > 2 \\
  A & x = 2 \\
  2x + 1 & 2 > x \geq 0 \\
  x^2 + 3 & x < 0 
\end{cases} \)
a) For what value of \( A \) is \( f \) continuous at \( x = 2 \)? Explain!
b) Find the following limits or write DNE if the limit doesn’t exist. Show all work.
\( \lim_{x \to 2} f(x), \quad \lim_{x \to -1} f(x), \quad \lim_{x \to 0} f(x), \quad \lim_{x \to (-1)} f(x) \)
b) Is \( f(x) \) differentiable at \( x = 0 \)?

6. Find the following derivatives from the definition:
a) \( f(x) = x^2 + 3x \)  
b) \( g(x) = \frac{1}{x + 2} \)  
c) \( h(x) = \sqrt{x - 3} \)
7. The line \( y = 2x + 3 \) is tangent to the parabola \( y = x^2 + B \). Find \( B \).

8. Find the derivative of the following functions. Don’t simplify!
   a) \( f(x) = \frac{7}{x^{3/7}} + \sqrt{x^5} + x^7 + 45 \)
   b) \( g(x) = (x + 9) \ast (x^2 - 7x) \)
   c) \( h(x) = \left( \frac{x^2 + 7}{x^5 - 8x} \right)^9 \)
   d) \( k(x) = \frac{(x^4 + 2)^6}{\sqrt{x^3 + 5x}} \)
   e) \( \ln x^5 + x - \ln x \)

9. Find the equation of the tangent line for the graph of \( f(x) = 2\sqrt{x} + x^2 - 5 \) at \( x = 1 \).

10. Let \( f(x) = g(\sqrt{x} + 3) \). Find \( f(6) \) and \( f'(6) \).
    It is impossible to find \( g \), but it will be useful to use some of the following known values of \( g(x) \) and \( g'(x) \):
    \[ g(1) = 2, \quad g(2) = 5, \quad g(3) = 7, \quad g(4) = 2, \quad g(5) = 11, \quad g(6) = 13 \quad \text{and} \quad g(7) = 21 \]
    \[ g'(1) = 3, \quad g'(2) = 2, \quad g'(3) = 8, \quad g'(4) = 10, \quad g'(5) = 12, \quad g'(6) = 21 \quad \text{and} \quad g'(7) = 23. \]

11. Sketch a possible graph of \( F \) on \([-3, 3]\) such that:
    \( F \) is continuous on \([-3, 0) \) and \((0, 3]\), \( \lim_{x \to 0^+} F(x) = 5 \), \( \lim_{x \to 0^-} F(x) = -2 \) \( F \) is not differentiable only at \( x = 0 \) and \( x = 1 \).

12. Let \( y = 2x^4 + 3x^2 + 12 \). Find \( \frac{d^3y}{dx^3} \).

13. The distance \( s \) (in feet) covered by a car \( t \) seconds after starting from rest is given by \( s(t) = 20t + 6t^2 + t^3 \), when \( 0 \leq t \leq 20 \).
   a) What is the velocity of the car 5 seconds after starting from rest?
   b) What is the acceleration of the car at that time?

14. Sketch a possible function on the domain \((-2, 4)\) that is:
    Not differentiable only at \( x = (-1.5) , \quad (-1) , \quad 0 , \quad 0.5 , \quad 1 , \quad 3 \), not continuous only at \( x = (-1) , \quad 0.5 , \quad 3 \) and has no limit only at \( x = 0.5 , \quad 3 \).

15. Let \( h(x) = f(g(x)) \). Assume that \( f(1) = 2 \), \( f'(1) = 7 \), \( f(2) = 5 \), \( f'(2) = 5 \), \( g(1) = 2 \) and \( g'(1) = 3 \). Find \( h'(1) \), \( (fg)'(1) \), \( (f/g)(1) \).
16. Suppose that $f$ and $g$ are differentiable functions such that $f(g(x)) = 8x^2$ to all real numbers $x$. Assume that $f(2) = 7$, $g(2) = 4$, $f'(2) = 4$ and $f'(4) = 2$. What is $g'(2)$?

17. Give an exact solution for the following population growth problem: The number of students taking 135 increased from 2,300 to 3,200 in the last 2 years. Assuming that this is an exponential growth, find $K$, the growth constant and use it to find the number of 135 students 3 years from now.

18. Expand the following expression: $\ln \frac{\sqrt{x}x^3y^2}{6\sqrt{y}x^9}$