1. If \( f(x) = \frac{1}{x} \), find the number \( c \) promised by the Mean Value Theorem on the interval \([1, 3]\). Don’t forget to check the hypotheses of the MVT!

\[
\begin{array}{c|c}
c & \sqrt{3} \\
\end{array}
\]

**MVT** If \( f(x) \) is continuous on \([a, b]\) and differentiable on \((a, b)\), then there is a \( c \) in \((a, b)\) so that

\[
f'(c) = \frac{f(b) - f(a)}{b - a}.
\]

\[
f'(x) = \frac{-1}{x^2},
\]

\[
-\frac{1}{c^2} = \frac{\frac{1}{3} - 1}{3 - 1} = \frac{\frac{2}{3}}{2} = -\frac{1}{3}.
\]

\[
c^2 = 3 \Rightarrow c = \sqrt{3}.
\]

\[
f'(c) = \frac{1}{\sqrt{3}} \text{ is differentiable on } (0, -).
\]

2. Find the equation of the tangent line to the curve \( x^2y^2 = x^2 + 2y^2 + 14 \) at \((2, 3)\).

**Equation:**

\[
\left( y - 3 \right) = -\frac{8}{3} \left( x - 2 \right)
\]

\[
2xy^2 + x^2y' + 2y^2y' = 2x + 2y y''
\]

\[
2\cdot2\cdot3^2 + 2^2\cdot2\cdot3y' = 2\cdot2 + 4\cdot3 y' \]

\[
3 \cdot 6 + 2\cdot4\cdot\frac{y'}{3} = 4 + 12y'
\]

\[
12y' = -32
\]

\[
y' = \frac{-32}{12} = -\frac{8}{3}
\]
3a. \[
\frac{d}{dx} \left( \frac{\tan 2x}{3x} \right) = \frac{(3x)\sec^2 2x \cdot 2 - (\tan 2x) \cdot 3}{(3x)^2} \cdot dx
\]

3b. If the cost of manufacturing \( q \) units of a product is \( C(q) = 3q^2 + q + 300 \), use marginal analysis to estimate the cost of producing the 17th item.

\[
\text{Cost} = 97
\]

\[
C'(q) = 6q + 1
\]

\[
C'(16) = 96 + 1 = 97
\]

4. a. Find \( \lim_{x \to \infty} \frac{e^{-2x} - 2}{e^{-3x} + 7} \).

\[
\lim_{x \to \infty} \frac{e^{-2x} - 2}{e^{-3x} + 7} = \frac{0 - 2}{0 + 7} = -\frac{2}{7}
\]

b. Find \( \lim_{x \to \infty} \frac{e^{-2x} - 2}{e^{-3x} + 7} \).

\[
\lim_{x \to \infty} \frac{e^{-2x} - 2}{e^{-3x} + 7} = 0
\]

\[
\lim_{x \to -\infty} \frac{e^{2x} - 2}{e^{-3x} + 7} = \frac{\infty}{\infty}
\]

\[
\lim_{x \to -\infty} \frac{-2e^{-2x}}{-3e^{-3x}} = \lim_{x \to -\infty} \frac{2}{3} \cdot \frac{e^x}{0} = 0
\]
5. Find the absolute maximum and minimum of the function \( f(x) = \frac{4x}{x^2 + 4} \) on the interval \([1, 10]\). Please give both \( x \) and \( y \) values.

<table>
<thead>
<tr>
<th>Absolute maximum:</th>
<th>((2, 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute minimum:</td>
<td>((10, \frac{10}{104}))</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{\text{f}'(x)}{x^2+4} & = \frac{(x^2+4)(4) - (4x)(2x)}{(x^2+4)^2} = \frac{-4x^2 + 16}{(x^2+4)^2} \\
\text{Crit. } x = \pm 2 \quad \text{Only } x = 2 \text{ is in } [1, 10] \ .
\end{align*}
\]

\[
\begin{align*}
\text{f}(1) & = \frac{4}{5} = 0.8 \\
\text{f}(2) & = \frac{8}{8} = 1 \\
\text{f}(10) & = \frac{40}{104} < 0.4
\end{align*}
\]

\[
\text{f}(x) = \frac{4x}{x^2+4} \quad \text{is a rational function that is defined everywhere, so it is continuous for all values of } x .
\]

This part was not required, but it's not a bad idea to think it through.
6. The function \( f(x) = (2x - 1)e^{4x} \) has \( f'(x) = (8x - 2)e^{4x} \) and \( f''(x) = 32xe^{4x} \). Find the intervals where \( f \) is increasing and decreasing and concave up and concave down. Find the \( x \)-coordinates of all relative extrema.

<table>
<thead>
<tr>
<th>Increasing:</th>
<th>((0, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decreasing:</td>
<td>((-\infty, \frac{1}{4}))</td>
</tr>
<tr>
<td>Concave up:</td>
<td>((0, \infty))</td>
</tr>
<tr>
<td>Concave down:</td>
<td>((-\infty, 0))</td>
</tr>
<tr>
<td>Relative Maxima:</td>
<td>None</td>
</tr>
<tr>
<td>Relative Minima:</td>
<td>(\frac{1}{4})</td>
</tr>
</tbody>
</table>


7. At noon, a flatbed truck leaves Winslow, Arizona, traveling north at 65 miles per hour. At 2 pm, a Volkswagen bus leaves the same corner traveling west at 60 miles per hour. How fast is the distance between the two vehicles changing at 5 pm? You do not need to multiply out any big numbers.

\[
\text{Rate} = \frac{180(60) + 325(65)}{\sqrt{180^2 + 325^2}}
\]

\[
x^2 + y^2 = z^2
\]

\[
\frac{dx}{dt} + \frac{dy}{dt} = z \frac{dz}{dt}
\]

\[
\frac{dz}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}
\]

\[
\Delta t = 5 \text{ pm}, \ x = 180, \ y = 325
\]

\[
\frac{dx}{dt} = 60, \ \frac{dy}{dt} = 65
\]

\[
z = \sqrt{180^2 + 325^2}
\]
8. A rectangular parcel has a square base of side \( x \) and a third side of length \( y \). Postal regulations say that the perimeter of the square plus the length of side \( y \) cannot exceed 102. Find the volume of the largest parcel allowed by these regulations.

\[
4x + y = 102
\]

\[
y = 102 - 4x
\]

\[
V = x^2 y = x^2 (102 - 4x)
\]

\[
= 102x^2 - 4x^3
\]

\[
\frac{dV}{dx} = 204x - 12x^2 = 12x (17 - x)
\]

Crit #3 \( x = 0, x = 17 \quad 0 \leq x \leq \frac{102}{4} \)

\[
f(0) = 0
\]

\[
f(17) = 102 \cdot 17^2 - 4 \cdot 17^3 = 17^2 (102 - 4 \cdot 17)
\]

\[
= 17^2 (102 - 68)
\]

\[
= 17^2 \cdot 34
\]

Volume = 17^2 \cdot 34
9. Let \( f(x) = \frac{2x + 5}{4x + 2} \). Find intervals where \( f \) is increasing and decreasing, and concave up and concave down. Find all horizontal and vertical asymptotes and find all relative maxima, minima, and inflections. Sketch the graph for 1 pt extra credit.

<table>
<thead>
<tr>
<th>Intervals where increasing:</th>
<th>NONE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intervals where decreasing:</td>
<td>((-\infty, -2) \cup (-2, \infty))</td>
</tr>
<tr>
<td>Intervals where concave up:</td>
<td>((-\frac{1}{2}, \infty))</td>
</tr>
<tr>
<td>Intervals where concave down:</td>
<td>((-\infty, -\frac{1}{2}))</td>
</tr>
<tr>
<td>Horizontal asymptotes:</td>
<td>(y = \frac{1}{2})</td>
</tr>
<tr>
<td>Vertical asymptotes:</td>
<td>(x = -\frac{1}{2})</td>
</tr>
<tr>
<td>Inflections:</td>
<td>NONE</td>
</tr>
<tr>
<td>Relative maxima:</td>
<td>NONE</td>
</tr>
<tr>
<td>Relative minima:</td>
<td>NONE</td>
</tr>
</tbody>
</table>

\[
f'(x) = \frac{(4x + 2) \cdot 2 - (2x + 5) \cdot 4}{(4x + 2)^2} = \frac{4 - 20}{(4x + 2)^2} = -\frac{16}{(4x + 2)^2}
\]

\[
f''(x) = \frac{(-16)(-2)}{(4x + 2)^3} \cdot 4 = \frac{128}{(4x + 2)^3}
\]

No unit #’s
10a. If \( y = (\cos x)^{\sin x} \), find \( y' \).

\[
y' = \left( \cos x \right)^{\sin x} \left[ \cos x \ln(\cos x) - \frac{\sin^2 x}{\cos x} \right]
\]

\[
\ln y = \ln \left( (\cos x)^{\sin x} \right) = \sin x \ln(\cos x)
\]

\[
\frac{1}{y} y' = \cos x \ln(\cos x) + \sin x \cdot \frac{1}{\cos x} \cdot (-\sin x)
\]

\[
y' = \left( \cos x \right)^{\sin x} \left[ \cos x \ln(\cos x) - \frac{\sin^2 x}{\cos x} \right]
\]

10b. Use linear approximation or differentials to estimate \( \sqrt{3.97} \).

\[
L(x) = f(a) + f'(a)(x-a)
\]

\[
f(x) = \sqrt{x} \quad a = 4, \ x = 3.97
\]

\[
f'(x) = \frac{1}{2} x^{-\frac{1}{2}}
\]

\[
f(4) = 2
\]

\[
f'(4) = \frac{1}{4}
\]

\[
L(3.97) = 2 + \frac{1}{4} (3.97 - 4)
\]

\[
= 2 + \frac{-0.03}{4}
\]

\[
= 2 - 0.0075
\]