Practice Midterm 1

The purpose of this practice midterm is to show you how well prepared you are for the first exam. The difficulty of this practice midterm is similar to the difficulty of the more challenging 50 percent of your first midterm exam. However, the questions on this practice midterm are not very similar to questions on the first midterm exam. In other words, the actual midterm exam is NOT the practice midterm “with the numbers changed”. The actual midterm exam may cover topics that are not represented on this practice midterm exam. The exams are closed book and we DO NOT allow notes, crib sheets, calculators of any type and electronic devices during exams. For this reason, you should attempt these practice midterm problems without any aids like books, notes or electronics. The solutions to these problems will be posted soon. By the way, you must show all work on exams. The exams are NOT multiple choice. When you take the exam, you will see blank space under each question. You will write your answer in that blank space, showing all work.

(1) Evaluate \( \int \sin^5 x \cos^4 x \, dx \), \( \int \frac{x^3 \, dx}{\sqrt{9 + x^2}} \).

(2) Evaluate \( \int e^{2x} \cos(3x) \, dx \) using two integrations by parts.

(3) Evaluate \( \int (\ln x)^2 \, dx \), \( \int \frac{4x^2 + 5x + 2}{x^3 - x^2} \, dx \), \( \int_1^{\infty} \frac{dx}{10 - 2x + x^2} \).

(4) Using the Method of Disks, find the volume of a cone with height \( H \) and circular base with radius \( R \).

(5) Using the Method of Shells, find the volume of a cone with height \( H \) and circular base with radius \( R \). Of course, you should get the same answer that you got in problem (4).

(6) Prove the reduction formula \( \int \sinh^n x \, dx = \frac{\sinh^{n-1} x \cosh x}{n} - \frac{n - 1}{n} \int \sinh^{n-2} x \, dx \) using integration by parts.

(7) Evaluate \( \int x^3 \cos(\ln x) \, dx \) using two integrations by parts.

(8) Prove that \( \int_1^{\infty} e^{-x^2} \, dx \) converges.

(9) Find the average value of \( f(x) = x \sin^{-1} x \) on \([0, 1]\). Note that \( \sin^{-1} \) is \( \text{arcsin} \).

(10) Let \( \mathcal{R} \) be the region in the \( xy \)-plane that is bounded by \( y = 4 - x^2 \) and \( y = 1 + x^2 \). Find the volume of the solid that is obtained by rotating \( \mathcal{R} \) about the \( x \)-axis.