(1) Simplify \( \sin^{-1}(\sin(7\pi/4)) \) and \( \sin^{-1}(\sin(5\pi/6)) \).

Recall that the formula \( \sin^{-1}(\sin(x)) = x \) is valid only when \(-\pi/2 \leq x \leq \pi/2\).

\[
\sin(7\pi/4) = \sin(7\pi/4 - 2\pi) = \sin(-\pi/4) \quad \text{and} \quad -\pi/2 < -\pi/4 < \pi/2.
\]

Therefore,

\[
\sin^{-1}(\sin(7\pi/4)) = \sin^{-1}(\sin(-\pi/4)) = -\pi/4.
\]

\[
\sin(5\pi/6) = -\sin(5\pi/6 - \pi) = -\sin(-\pi/6) = \sin(\pi/6) \quad \text{and} \quad -\pi/2 < \pi/6 < \pi/2.
\]

Therefore,

\[
\sin^{-1}(\sin(5\pi/6)) = \sin^{-1}(\sin(\pi/6)) = \pi/6.
\]

(2) Assume \( x_0 \) has the properties \( \sin(x_0) = 0.7 \) and \( \cos(x_0) < 0 \). Evaluate \( \sin(4x_0) \) and \( \cos(4x_0) \).

We know \( \cos(x_0) = \pm \sqrt{1 - \sin^2(x_0)} = \pm \sqrt{1 - (0.7)^2} = \pm \sqrt{0.51} \) and \( \cos(x_0) \) is negative. Therefore, the correct choice of \( \pm \) is \( - \) and \( \cos(x_0) = -\sqrt{0.51} \). Now

\[
\sin(2x_0) = 2 \sin(x_0) \cos(x_0) = 2(0.7)(-\sqrt{0.51}) = -(1.4)\sqrt{0.51}
\]

and

\[
\cos(2x_0) = \cos^2(x_0) - \sin^2(x_0) = 0.51 - (0.7)^2 = 0.02.
\]

Then

\[
\sin(4x_0) = 2 \sin(2x_0) \cos(2x_0) = 2(-1.4)\sqrt{0.51}(0.02) = -0.056\sqrt{0.51}
\]

and

\[
\cos(4x_0) = \cos^2(2x_0) - \sin^2(2x_0) = (0.02)^2 - (-1.4)\sqrt{0.51}
\]

\[
= 0.0004 - (1.96)(0.51) = -0.9992.
\]

(3) Simplify \( \cot(\sin^{-1} x) \) and \( \cos(\tan^{-1} x) \).

We know \( (\cos(\sin^{-1} x))^2 = \cos^2(\sin^{-1} x) = 1 - \sin^2(\sin^{-1} x) = 1 - (\sin(\sin^{-1} x))^2 = 1 - x^2 \), hence \( \cos(\sin^{-1} x) = \pm \sqrt{1 - x^2} \). Since \( \sin^{-1} x \) is in the interval \([-\pi/2, \pi/2]\), we must have \( \cos(\sin^{-1} x) \geq 0 \), and this implies that the \( \pm \) must be \( + \). This says \( \cos(\sin^{-1} x) = \sqrt{1 - x^2} \).

We also know \( \sin(\sin^{-1} x) = x \). This implies

\[
\cot(\sin^{-1} x) = \frac{\cos(\sin^{-1} x)}{\sin(\sin^{-1} x)} = \frac{\sqrt{1 - x^2}}{x}.
\]
We know \((\sec(\tan^{-1} x))^2 = \sec^2(\tan^{-1} x) = 1 + \tan^2(\tan^{-1} x) = 1 + (\tan(\tan^{-1} x))^2 = 1 + x^2)\), hence \(\sec(\tan^{-1} x) = \pm \sqrt{1 + x^2}\). Since \(\tan^{-1} x\) is in the interval \((-\pi/2, \pi/2)\), we must have \(\sec(\tan^{-1} x) > 0\), and this implies that the \pm must be +. This says \(\sec(\tan^{-1} x) = \sqrt{1 + x^2}\). This implies

\[
\cos(\tan^{-1} x) = \frac{1}{\sec(\tan^{-1} x)} = \frac{1}{\sqrt{1 + x^2}}.
\]

(4) Find all solutions of the equation \(\left(e^{6 + x^2/2}\right)^2 = 1\).

Since \((e^t)^2 = e^{2t}\), this can rewritten as \(e^{2(6 + x^2/2)} = e^{7x}\). Since \(e^u = e^v\) implies \(u = v\), we now know \(2(6 + x^2/2) = 7x\), which is \(x^2 - 7x + 12 = 0\). The quadratic equation gives us the solutions \(x = 3\) and \(x = 4\).

(5) Solve for \(x\) in the equation \(\log_{27} x = 1/3\).

From \(\log_{27} x = 1/3\) we get \(27^{1/3} = x\). Since \(27^{1/3} = 3\), we get \(x = 3\).

(6) The position of a particle at time \(t\) seconds is \(s = t^2 + 1/t\) meters. Find the average velocity over the time interval \([1, 5]\).

The average velocity over the time interval \([1, 5]\) is

\[
\frac{(5^2 + 1/5) - (1^2 + 1/1)}{5 - 1} = 5.8 \text{ meters/second}.
\]

(7) Find \(\lim_{x \to 5^+} \frac{x - 5}{|x - 5|}\) and \(\lim_{x \to 5^-} \frac{x - 5}{|x - 5|}\).

If \(x\) approaches 5 from the right then \(x > 5\) and \(x - 5 > 0\), which implies \(|x - 5| = x - 5\). This gives

\[
\lim_{x \to 5^+} \frac{x - 5}{|x - 5|} = \lim_{x \to 5^+} \frac{x - 5}{x - 5} = \lim_{x \to 5^+} 1 = 1.
\]

If \(x\) approaches 5 from the left then \(x < 5\) and \(x - 5 < 0\), which implies \(|x - 5| = -(x - 5)\). This gives

\[
\lim_{x \to 5^-} \frac{x - 5}{|x - 5|} = \lim_{x \to 5^-} \frac{x - 5}{-(x - 5)} = \lim_{x \to 5^-} -1 = -1.
\]

(8) Find \(\lim_{x \to 4^+} \frac{10 - x^2}{(x - 4)^5}\) and \(\lim_{x \to 4^-} \frac{10 - x^2}{(x - 4)^5}\).

As \(x\) approaches 4 from the right, \(x - 4\) approaches 0 and remains positive. This implies that \((x - 4)^5\) approaches 0 and remains positive as \(x\) approaches 4 from the right. This
implies \( \lim_{x \to 4^+} 1/(x - 4)^5 = \infty \). Also, \( \lim_{x \to 4^+} (10 - x^2) = -6 \). This shows \( \lim_{x \to 4^+} (10 - x^2)/(x - 4)^5 = (-6)\infty = -\infty \).

As \( x \) approaches 4 from the left, \( x - 4 \) approaches 0 and remains negative. This implies that \( (x - 4)^5 \) approaches 0 and remains negative as \( x \) approaches 4 from the left. This implies \( \lim_{x \to 4^-} 1/(x - 4)^5 = -\infty \). Also, \( \lim_{x \to 4^-} (10 - x^2) = -6 \). This shows \( \lim_{x \to 4^-} (10 - x^2)/(x - 4)^5 = (-6)(-\infty) = \infty \).

(9) A continuous function \( f \) is defined by

\[
f(x) = \begin{cases} 
x^2 + x & \text{if } x < a, \\
x^2 - 1 & \text{if } a \leq x < b, \\
9 - 4x^2 & \text{if } b \leq x,
\end{cases}
\]

where \( a \) and \( b \) are real numbers such that \( a < b \). Find \( a \) and \( b \).

Continuity at \( a \) implies \( \lim_{x \to a^-} f(x) = \lim_{x \to a^-} f(a) = a^2 - 1 \). The definition of \( f(x) \) also implies \( \lim_{x \to a^-} f(x) = \lim_{x \to a^-} x^2 + x = a^2 + a \). Combining these two assertions, we get \( a^2 - 1 = a^2 + a \), which gives \( a = -1 \). Similarly, continuity at \( b \) implies \( \lim_{x \to b^-} f(x) = \lim_{x \to b^-} f(x) = f(b) = 9 - 4b^2 \). The definition of \( f(x) \) also implies \( \lim_{x \to b^-} f(x) = \lim_{x \to b^-} x^2 - 1 = b^2 - 1 \).

Combining the last two assertions, we get \( 9 - 4b^2 = b^2 - 1 \), which gives \( 10 = 5b^2 \). This implies \( 2 = b^2 \) and \( b = \pm \sqrt{2} \). We already showed \( a = -1 \). We have the assumption \( a < b \). This gives \( -1 = a < b \), which tells us that \( b = -\sqrt{2} \) is impossible. But we have shown \( b = \pm \sqrt{2} \). This implies that the \( \pm \) is +, and we get \( b = \sqrt{2} \).

(10) Evaluate \( \lim_{x \to 2} \frac{x^3 - 8}{x - 2} \), \( \lim_{x \to 3} \frac{3 - x}{\sqrt{3x + 1} - \sqrt{10}} \), \( \lim_{x \to \infty} \frac{x}{\sqrt{3x^2 + 7}} \).

Factorization gives us

\[
\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(x^2 + 2x + 4)}{x - 2} = \lim_{x \to 2} x^2 + 2x + 4 = 12.
\]

Rationalization gives us

\[
\lim_{x \to 3} \frac{3 - x}{\sqrt{3x + 1} - \sqrt{10}} = \lim_{x \to 3} \frac{(3 - x)(\sqrt{3x + 1} + \sqrt{10})}{(\sqrt{3x + 1} - \sqrt{10})(\sqrt{3x + 1} + \sqrt{10})} = \lim_{x \to 3} \frac{(3 - x)(\sqrt{3x + 1} + \sqrt{10})}{3(3x + 1) - 10} = \lim_{x \to 3} \frac{(3 - x)(\sqrt{3x + 1} + \sqrt{10})}{3(x - 3)} = \lim_{x \to 3} \frac{-\sqrt{3x + 1} + \sqrt{10}}{3} = -\frac{2\sqrt{10}}{3}.
\]
As $x$ approaches $-\infty$, it becomes negative and we get $x = -|x| = -\sqrt{x^2}$. This gives

$$
\lim_{x \to -\infty} \frac{x}{\sqrt{3x^2 + 7}} = \lim_{x \to -\infty} \frac{-\sqrt{x^2}}{\sqrt{3x^2 + 7}} = \lim_{x \to -\infty} -\sqrt{\frac{x^2}{3x^2 + 7}} = -\sqrt{\frac{1}{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.
$$

(11) Prove $\lim_{x \to 0} (x \cos^3(1/x)) = 0$ using the Squeeze Theorem.

We know $|\cos^3(1/x)| = |\cos(1/x)|^3 \leq 1$ for all $x \neq 0$. This implies $|x \cos^3(1/x)| = |x| \cdot |\cos^3(1/x)| \leq |x|$ for all $x \neq 0$. Now we know

$$
|x \cos^3(1/x)| \leq |x| \text{ for } x \neq 0.
$$

This can be rewritten as

$$
-|x| \leq x \cos^3(1/x) \leq |x| \text{ for } x \neq 0
$$

because, in general, $|t| \leq c$ is equivalent to $-c \leq t \leq c$. Now the Squeeze Theorem and the facts $\lim_{x \to 0} -|x| = 0 = \lim_{x \to 0} |x|$ lead to the conclusion $\lim_{x \to 0} (x \cos^3(1/x)) = 0$.

(12) Evaluate $\lim_{x \to 0} \frac{\sin(7x)}{\tan(5x)}$ and $\lim_{x \to 0} \frac{x^2}{1 - \cos x}$.

Using the substitution $u = 7x$ we obtain

$$
\lim_{x \to 0} \frac{\sin(7x)}{x} = 7 \lim_{x \to 0} \frac{\sin(7x)}{7x} = 7 \lim_{u \to 0} \frac{\sin(u)}{u} = 7 \cdot 1 = 7.
$$

Using the substitution $v = 5x$ we obtain

$$
\lim_{x \to 0} \frac{\tan(5x)}{x} = 5 \lim_{x \to 0} \frac{\tan(5x)}{5x} = 5 \lim_{u \to 0} \frac{\tan(u)}{u} = 5 \lim_{u \to 0} \frac{\sin(u)}{u} \cdot \lim_{u \to 0} \cos u
$$

$$
= 5 \cdot 1 = 5.
$$

All this gives us

$$
\lim_{x \to 0} \frac{\sin(7x)}{\tan(5x)} = \lim_{x \to 0} \frac{\sin(7x)}{x} \cdot \lim_{x \to 0} \frac{x}{\tan(5x)} = \frac{7}{5}.
$$

From the fact $\lim_{x \to 0} \frac{\sin x}{x} = 1$ we get $\lim_{x \to 0} \frac{x}{\sin x} = \frac{1}{\lim_{x \to 0} \sin x} = \frac{1}{1} = 1$. This implies

$$
\lim_{x \to 0} \frac{x^2}{1 - \cos x} = \lim_{x \to 0} \left(\frac{x}{\sin x}\right)^2 = 1^2 = 1. \text{ Now we have}
$$

$$
\lim_{x \to 0} \frac{x^2}{1 - \cos x} = \lim_{x \to 0} \frac{x^2(1 + \cos x)}{(1 - \cos x)(1 + \cos x)} = \lim_{x \to 0} \frac{x^2(1 + \cos x)}{1 - \cos^2 x} = \lim_{x \to 0} \frac{x^2(1 + \cos x)}{\sin^2 x}
$$

$$
= \left(\lim_{x \to 0} \frac{x^2}{\sin^2 x}\right) \left(\lim_{x \to 0} (1 + \cos x)\right) = 1 \cdot (1 + 1) = 2.
$$
(13) Using the Intermediate Value Theorem, prove that the equation
\[ x^5 + x^4 - 7x^3 + 2x^2 + 3x + \pi = 0 \]
has a real solution \( x \).
Let \( f(x) = x^5 + x^4 - 7x^3 + 2x^2 + 3x + \pi \). Then \( f(x) \) is a continuous function such that \( f(-100) < 0 < f(100) \). The IVT tells us that there exists \( c \) in the interval \((-100, 100)\) such that \( f(c) = 0 \). Then \( x = c \) is a real solution of the equation \( x^5 + x^4 - 7x^3 + 2x^2 + 3x + \pi = 0 \).

(14) Prove \( \lim_{x \to 4} (3x - 5) = 7 \) using an \( \varepsilon-\delta \) argument.

Let \( \varepsilon > 0 \) be given. Choose \( \delta = \varepsilon/3 \). We know \( \delta > 0 \). If \( 0 < |x - 4| < \delta \) then 
\[ |(3x - 5) - 7| = |3x - 12| = |3(x - 4)| = 3 \cdot |x - 4| < 3\delta = 3(\varepsilon/3) = \varepsilon. \]

(15) Prove \( \lim_{x \to 0} \sqrt{|x|} = 0 \) using an \( \varepsilon-\delta \) argument.

Let \( \varepsilon > 0 \) be given. Choose \( \delta = \varepsilon^2 \). We know \( \delta > 0 \). If \( 0 < |x - 0| < \delta \) then \( 0 < |x| < \delta \), hence 
\[ |\sqrt{|x|} - 0| = \sqrt{|x|} < \sqrt{\delta} = \sqrt{\varepsilon^2} = \varepsilon. \]

(16) Assume \( f(x) = \sqrt{x} \). Prove \( f'(x) = \frac{1}{2\sqrt{x}} \) using the definition
\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]
of the derivative.

We see
\[
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{(\sqrt{x + h} - \sqrt{x})(\sqrt{x + h} + \sqrt{x})}{h(\sqrt{x + h} + \sqrt{x})} \\
= \lim_{h \to 0} \frac{(x + h) - x}{h(\sqrt{x + h} + \sqrt{x})} = \lim_{h \to 0} \frac{h}{h(\sqrt{x + h} + \sqrt{x})} \\
= \lim_{h \to 0} \frac{1}{\sqrt{x + h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.
\]

(17) Assume \( f(x) = \frac{1}{\sqrt{x}} \). Prove \( f'(x) = -\frac{1}{2x^{3/2}} \) using the definition
\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]
of the derivative.
We see
\[
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{h} \cdot \left( \frac{1}{\sqrt{x + h}} - \frac{1}{\sqrt{x}} \right) = \lim_{h \to 0} \frac{1}{h} \cdot \frac{\sqrt{x} - \sqrt{x + h}}{\sqrt{x} \sqrt{x + h}}
\]
\[
= \lim_{h \to 0} \frac{1}{h} \cdot \frac{(\sqrt{x} - \sqrt{x + h})(\sqrt{x} + \sqrt{x + h})}{\sqrt{x} \sqrt{x + h}(\sqrt{x} + \sqrt{x + h})}
\]
\[
= \lim_{h \to 0} \frac{1}{h} \cdot \frac{x - (x + h)}{\sqrt{x} \sqrt{x + h}(\sqrt{x} + \sqrt{x + h})}
\]
\[
= \lim_{h \to 0} \frac{1}{h} \cdot \frac{-h}{\sqrt{x} \sqrt{x + h}(\sqrt{x} + \sqrt{x + h})}
\]
\[
= \lim_{h \to 0} \frac{1}{\sqrt{x} \sqrt{x + h}(\sqrt{x} + \sqrt{x + h})} = \frac{1}{2x^{3/2}}.
\]

(18) Evaluate the following derivatives:
\[
\frac{d}{dx} \sin^{-1}(e^x), \quad \frac{d}{dx} \tan^{-1}(\sqrt{x}), \quad \frac{d}{dx} \sqrt{1 + \sqrt{x}},
\]
\[
\frac{d}{dx} \left( x^2 e^{\cos x} \right), \quad \frac{d}{dx} \left( \sec^{-1}(x) \right)^3, \quad \frac{d}{dx} \tan(x^2 + x^4).
\]

Using the Chain Rule and other Rules, we get
\[
\frac{d}{dx} \sin^{-1}(e^x) = \frac{e^x}{\sqrt{1 - (e^x)^2}} = \frac{e^x}{\sqrt{1 - e^{2x}}},
\]
\[
\frac{d}{dx} \tan^{-1}(\sqrt{x}) = \frac{(1/2)x^{-1/2}}{1 + (\sqrt{x})^2} = \frac{(1/2)x^{-1/2}}{1 + x},
\]
\[
\frac{d}{dx} \sqrt{1 + \sqrt{x}} = \frac{1}{2}(1 + \sqrt{x})^{-1/2} \frac{1}{2\sqrt{x}},
\]
\[
\frac{d}{dx} \left( x^2 e^{\cos x} \right) = 2xe^{\cos x} + x^2(-\sin x)e^{\cos x} = (2x - x^2 \sin x)e^{\cos x},
\]
\[
\frac{d}{dx} \left( \sec^{-1}(x) \right)^2 = 2 \sec^{-1}(x) \cdot \frac{1}{|x|\sqrt{x^2 - 1}},
\]
\[
\frac{d}{dx} \tan(x^2 + x^4) = \frac{\sec^2(x^2 + x^4)(2x + 4x^3)(1 + x^6) - 6x^5 \tan(x^2 + x^4)}{(1 + x^6)^2}.
\]
(19) Find the second derivative of each of the following functions:

\[ \sin^5 x, \quad \tan(x), \quad e^{x^4 + x}, \quad \sin^{-1}(x^2), \quad \frac{1}{4 + 5x^2}. \]

If \( f(x) = \sin^5 x \) then \( f'(x) = 5 \sin^4 x \cos x \) and

\[ f''(x) = 5(4 \sin^3 x \cos x)(\cos x) + 5 \sin^4 x(- \sin x) = 20 \sin^3 x \cos^2 x - 5 \sin^5 x. \]

If \( f(x) = \tan(x) \) then \( f'(x) = \sec^2(x)(1/2)x^{-1/2} = (1/2) \sec^2(x)x^{-1/2} \) and

\[ f''(x) = (1/2)2 \sec(x) \sec(x) \tan(x)(1/2)x^{-1/2}x^{-1/2} + (1/2) \sec^2(x)(-1/2)x^{-3/2}. \]

If \( f(x) = e^{x^4 + x} \) then \( f'(x) = (4x^3 + 1)e^{x^4 + x} \) and

\[ f''(x) = 12x^2e^{x^4 + x} + (4x^3 + 1)(4x^3 + 1)e^{x^4 + x} = (12x^2 + (4x^3 + 1)^2)e^{x^4 + x}. \]

If \( f(x) = \sin^{-1}(x^2) \) then \( f'(x) = \frac{2x}{\sqrt{1 - (x^2)^2}} = \frac{2x}{\sqrt{1 - x^4}} = 2x(1 - x^4)^{-1/2} \) and

\[ f''(x) = 2(1 - x^4)^{-1/2} + 2x(-1/2)(1 - x^4)^{-3/2}(-4x^3). \]

If \( f(x) = \frac{1}{4 + 5x^2} \) then \( f(x) = (4 + 5x^2)^{-1} \), hence

\[ f'(x) = (-1)(4 + 5x^2)^{-2}10x = -10x(4 + 5x^2)^{-2} \]

and

\[ f''(x) = -10(4 + 5x^2)^{-2} - 10x(-2)(4 + 5x^2)^{-3}10x. \]

(20) Find the slope of the tangent line at the point \((1, 2)\) on the graph of \(x^2y^4 + xy = 18\).

Implicit differentiation gives

\[ 2xy^4 + x^2 \frac{dy}{dx} + y + x \frac{dy}{dx} = 0. \]

Substituting \( x = 1 \) and \( y = 2 \) gives

\[ 2 \cdot 2^4 + 4 \cdot 2^3 \frac{dy}{dx} + 2 + \frac{dy}{dx} = 0. \]

Therefore, \( \frac{dy}{dx} = -\frac{34}{33} \) when \( x = 1 \) and \( y = 2 \). The slope of the tangent line at the point \((1, 2)\) on the graph of \(x^2y^4 + xy = 18\) is \( -\frac{34}{33} \).