Review Sheet for Exam 1, Math 151, Fall 2017

These problems are presented in order to help you understand the material that is listed prior to the first exam in the syllabus. DO NOT assume that your first midterm exam will resemble this set of problems. The following 20 problems are not meant to be a sample exam. These problems are just a study aid. Since we have not covered L'Hôpital's Rule yet, this rule should not be used to answer any of these questions.

(1) Simplify $\sin^{-1}(\sin(7\pi/4))$ and $\sin^{-1}(\sin(5\pi/6))$.

(2) Assume x_0 has the properties $\sin(x_0) = 0.7$ and $\cos(x_0) < 0$. Evaluate $\sin(4x_0)$ and $\cos(4x_0)$.

(3) Simplify $\cot(\sin^{-1} x)$ and $\cos(\tan^{-1} x)$.

(4) Find all solutions of the equation $\dfrac{\left(e^{6+x^2/2}\right)^2}{e^{7x}} = 1$.

(5) Solve for x in the equation $\log_{27} x = 1/3$.

(6) The position of a particle at time t seconds is $s = t^2 + 1/t$ meters. Find the average velocity over the time interval $[1, 5]$.

(7) Find $\displaystyle\lim_{x\to 5^+} \frac{x-5}{|x-5|}$ and $\displaystyle\lim_{x\to 5^-} \frac{x-5}{|x-5|}$.

(8) Find $\displaystyle\lim_{x\to 4^+} \frac{10-x^2}{(x-4)^5}$ and $\displaystyle\lim_{x\to 4^-} \frac{10-x^2}{(x-4)^5}$.

(9) A continuous function f is defined by

$$f(x) = \begin{cases} x^2 + x & \text{if } x < a, \\ x^2 - 1 & \text{if } a \le x < b, \\ 9 - 4x^2 & \text{if } b \le x, \end{cases}$$

where a and b are real numbers such that $a < b$. Find a and b.

(10) Evaluate $\displaystyle\lim_{x\to 2} \frac{x^3-8}{x-2}$, $\displaystyle\lim_{x\to 3} \frac{3-x}{\sqrt{3x+1}-\sqrt{10}}$, $\displaystyle\lim_{x\to -\infty} \frac{x}{\sqrt{3x^2+7}}$.

(11) Prove $\displaystyle\lim_{x\to 0}(x\cos^3(1/x)) = 0$ using the Squeeze Theorem.

(12) Evaluate $\displaystyle\lim_{x\to 0} \frac{\sin(7x)}{\tan(5x)}$ and $\displaystyle\lim_{x\to 0} \frac{x^2}{1-\cos x}$.

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(13) Using the Intermediate Value Theorem, prove that the equation

$$x^5 + x^4 - 7x^3 + 2x^2 + 3x + \pi = 0$$

has a real solution x.

(14) Prove $\lim_{x \to 4} (3x - 5) = 7$ using an ε-δ argument.

(15) Prove $\lim_{x \to 0} \sqrt{|x|} = 0$ using an ε-δ argument.

(16) Assume $f(x) = \sqrt{x}$. Prove $f'(x) = \frac{1}{2\sqrt{x}}$ using the definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

of the derivative.

(17) Assume $f(x) = \frac{1}{\sqrt{x}}$. Prove $f'(x) = -\frac{1}{2x^{3/2}}$ using the definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

of the derivative.

(18) Evaluate the following derivatives:

$$\frac{d}{dx} \sin^{-1}(e^x) \ , \quad \frac{d}{dx} \tan^{-1}(\sqrt{x}) \ , \quad \frac{d}{dx} \sqrt{1 + \sqrt{x}} \ ,$$

$$\frac{d}{dx} \left(x^2 e^{\cos x}\right) \ , \quad \frac{d}{dx} \left(\sec^{-1}(x)\right)^3 \ , \quad \frac{d}{dx} \frac{\tan(x^2 + x^4)}{1 + x^6} \ .$$

(19) Find the second derivative of each of the following functions:

$$\sin^5 x \ , \quad \tan(\sqrt{x}) \ , \quad e^{x^4 + x} \ , \quad \sin^{-1}(x^2) \ , \quad \frac{1}{4 + 5x^2} \ .$$

(20) Find the slope of the tangent line at the point $(1, 2)$ on the graph of $x^2 y^4 + xy = 18$.