Abstract:

An \((\alpha, \beta)\)-spanner of a graph \(G\) is a subgraph \(H\) such that \(d_H(u, w) \leq \alpha \cdot d_G(u, w) + \beta\) for every pair of vertices \(u, w\), where \(d_G(u, w)\) denotes the distance between two vertices \(u\) and \(v\) in \(G\). It is known that every graph \(G\) has a polynomially constructible \((2\kappa - 1, 0)\)-spanner (a.k.a. multiplicative \((2\kappa - 1)\)-spanner) of size \(O(n^{1+1/\kappa})\) for every integer \(\kappa \geq 1\), and a polynomially constructible \((1, 2)\)-spanner (a.k.a. additive 2-spanner) of size \(O(n^{3/2})\). This paper explores hybrid spanner constructions (involving both multiplicative and additive factors) for general graphs and shows that the multiplicative factor can be made arbitrarily close to 1 while keeping the spanner size arbitrarily close to \(O(n)\), at the cost of allowing the additive term to be a sufficiently large constant. More formally, we show that for any constant \(\epsilon, \delta > 0\) there exists a constant \(\beta = \beta(\epsilon, \delta)\) such that for every \(n\)-vertex graph \(G\) there is an efficiently constructible \((1 + \epsilon, \beta)\)-spanner of size \(O(n^{1+\delta})\). It follows that for any constant \(\epsilon, \delta > 0\) there exists a constant \(\beta(\epsilon, \delta)\) such that for any \(n\)-vertex graph \(G = (V, E)\) there exists an efficiently constructible subgraph \((V, H)\) with \(O(n^{1+\delta})\) edges such that \(d_H(u, w) \leq (1 + \epsilon)d_G(u, w)\) for every pair of vertices \(u, w \in V\) such that \(d_G(u, w) \geq \beta(\epsilon, \delta)\). The talk is based on a joint paper with David Peleg, that appeared at STOC 2001.