Suggested problems on inequalities

1. Prove that for any real number $x \geq -1$, and any positive integer $n$ $(1 + x)^n \geq 1 + nx$.
2. Prove that $n! \geq (n/e)^n$ and that $n! \leq (n + 1) \left(\frac{n+1}{e}\right)^n$.
3. Suppose that $a_1, a_2, \ldots$ is a sequence of positive real numbers such that $\sum_{n=1}^{\infty} a_n$ converges. Prove that for any $p > 1/2$, $\sum_{n=1}^{\infty} \frac{a_n^p}{n^p}$ also converges.
4. Let $a_1, a_2, \ldots, a_n$ be positive real numbers and let $s$ denote their sum. Show that $(1 + a_1)(1 + a_2) \cdots (1 + a_n) \leq \sum_{i=0}^{n} s^i/i!$.
5. Let $p_1, \ldots, p_n$ be distinct points in the closed unit disc in the plane. Let $d_k$ be the distance from $p_k$ to the nearest other point. Show that $\sum_{k=1}^{n} (d_k)^2 \leq 16$.
6. If $a, b, c$ are positive reals with $abc = 1$, show that:
   \[ \frac{1}{a^3(b + c)} + \frac{1}{b^3(c + a)} + \frac{1}{c^3(a + b)} \geq \frac{3}{2} \]

11. If $a, b, c$ are positive reals, show that:
   \[ \frac{1}{a(1 + b)} + \frac{1}{b(1 + c)} + \frac{1}{c(1 + a)} \geq \frac{3}{1 + abc} \]