1. A frog hops on the number line from 0 to \( n \). If he is on position \( i \), then he can either hop to \( i + 1 \) or to \( i + 2 \). Let \( F_n \) be the number of ways he can get from 0 to \( n \). Show that \( F_n = F_{n-1} + F_{n-2} \).

2. How many diagonals does a regular \( n \)-gon have? How many triangles are there such that each edge of the triangle is a diagonal of the regular \( n \)-gon? In particular the edges of the triangle shouldn’t be edges of the \( n \)-gon.

3. There are 25 people in a committee. They form various subcommittees. Each subcommittee has exactly 4 members. Every pair of people are together in exactly one subcommittee. How many subcommittees were formed?

4. \( n \) letters are put in \( n \) addressed envelopes. Let \( d_n \) be the number of ways this can be done such that no letter goes into its designated envelope. Show that \( d_n = (n + 1)(d_{n-1} + d_{n-2}) \).

5. \( L \) is a set of \( n \) lines in \( \mathbb{R}^2 \). \( P \) is a set of \( n \) points in \( \mathbb{R}^2 \). For each point \( p \in P \), let \( d(p) \) be the number of lines in \( L \) passing through \( p \). Show that

\[
\sum_{p \in P} \binom{d(p)}{2} \leq \binom{n}{2}.
\]

Bonus: Use this to show that

\[
\sum_{p \in P} d(p) \leq \frac{n + n\sqrt{4n - 3}}{2}.
\]

6. Prove that

\[
\sum_{i=0}^{n} \binom{n}{i} \binom{2n}{i} = \binom{3n}{n}.
\]

7. By interpreting both sides of the equation as “the number of ways of ____”, show that

\[
\sum_{k=0}^{n} \binom{n}{k} 2^k = 3^n.
\]

8. How many positive integers less than 10000 are relatively prime to 10000?

9. How many positive integers less than 10000 are relatively prime to 30?

10. Show that

\[
n! = n^n - \binom{n}{1}(n-1)^n + \binom{n}{2}(n-2)^n \ldots + (-1)^i \binom{n}{i}(n-i)^n + \ldots + (-1)^n \binom{n}{n}(n-n)^n.
\]
11. Show that
\[
\binom{n}{0} + \binom{n + 1}{1} + \binom{n + 2}{2} + \ldots + \binom{n + k}{k} = \binom{n + k + 1}{k}.
\]
(You may either do this directly using what you know about binomial coefficients, or by 2-way counting).

12. How many \(n\) letter strings can you form that uses each of the 26 letters of the alphabet at least once?