**Problems**

1. How many 0's does 400! end with?
2. Find the smallest positive integer having exactly 100 positive divisors.
3. Define $a_1 = 1$, and for every $n > 1$, define $a_{n+1} = a_n + \frac{1}{a_n}$. Prove that $20 < a_{200} < 24$.
4. If $S$ is a set of real numbers let $S + S$ be the set of all sums of the form $a + b$ where $a \in S$ and $b \in S$ (where $a, b$ are allowed to be the same number). For each positive integer $n$, how should you choose $S$ of size $n$ if you want to minimize the size of $S + S$?
5. Among all lists of positive integers that sum to 1000, what is the most their product can be?
6. Find all integers $a, b$ such that $a^b = b^a$.
7. Determine all polynomials $P(x)$ that satisfy the equations $P(x^2+1) = (P(x))^2+1$ and $P(0) = 0$.
8. Let us say that a positive real number $x$ is a *Fermat number* if there are three distinct positive integers $a, b, c$ such that $a^x + b^x = c^x$. Prove that there exist arbitrarily large Fermat numbers (which means that for every real number $M$ there is a fermat number $x$ that is bigger than $M$).
9. Suppose we define the *linear size* of a box $B$, denote $LS(B)$ to be the sum of its length, width and height. Is it possible to construct two boxes $B, C$ where $LS(B) < LS(C)$ and $C$ fits inside of $B$?
10. Given 6 points in the plane, show that the ratio of the maximum distance between two of them to the minimum distance between two of them is at least $\sqrt{3}$.
11. Suppose we color the points of the $x$–$y$ plane with three colors. Prove that there must be two points at distance one from each other that get the same color.
12. Evaluate $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdots}}}$. (More formally, let $x_1 = \sqrt{2}$, and let $x_{n+1} = \sqrt{2}^{x_n}$ for each $n \geq 1$. Find $\lim_{n\to\infty} x_n$, if it exists.)
13. Standard U.S. coins are pennies (1 cent), nickels (5 cents), dimes (10 cents), quarters (25 cents), half dollars (50 cents) and dollars (100 cents). Say that an integer $k$ is *convertible* provided that it is possible to find a collection of $k$ coins that adds up to a dollar. What is the smallest positive integer that is not convertible?
14. Evaluate the determinant of the $n \times n$ matrix whose $(i, j)$th entry is $a^{|i-j|}$.
15. For a nonnegative integer $n$, let $f(n)$ be the number of ways to express $n$ as a sum of powers of 2, where each power of 2 is used at most twice. For example, $f(6) = 3$ since $6 = 4 + 2 = 4 + 1 + 1 = 2 + 2 + 1 + 1$. (We define $f(0) = 1$). For a positive integer $n$. let $r(n) = f(n)/f(n-1)$. The function $r$ is a rather amazing function: it is a bijection from the set of positive integers to the set of the positive rational numbers! Prove this.