Representation Theory and Mathematical Physics

Conference in Honor of Gregg Zuckerman's 60th Birthday
October 24–27, 2009
Yale University

Jeffrey Adams
Bong Lian
Siddhartha Sahi
Editors
Representation Theory and Mathematical Physics
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Gaitsgory, Dennis (Harvard)
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Dedicated to Gregg Zuckerman on the occasion of his 60th birthday.
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Preface

Lie groups and their representations are a fundamental area of mathematics, with connections to geometry, topology, number theory, physics, combinatorics, and many other areas. Gregg Zuckerman’s work lies at the very heart of the modern theory of representations of Lie groups. His influential ideas on derived functors, the translation principle, and coherent continuation laid the groundwork of modern algebraic theory.

Zuckerman has long been active in the fruitful interplay between mathematics and physics. Developments in this area include work on chiral algebras, and the representation theory of affine Kac-Moody algebras. Recent progress on the geometric Langlands program points to exciting connections between automorphic representations and dual fibrations in geometric mirror symmetry.

These topics were the subject of a conference in honor of Gregg Zuckerman’s 60th birthday, held at Yale, October 24-27, 2009.

Summary of Contributions

The classical Plancherel theorem is a statement about the Fourier transform on $L^2(\mathbb{R})$. It has generalizations to any locally compact group. The Plancherel Formula, The Plancherel Theorem, and the Fourier Transform of Orbital Integrals by Rebecca A. Herb and Paul J. Sally, Jr. surveys the history of this subject for non-abelian Lie groups and p-adic groups.

One of Zuckerman’s major contributions to representation theory is the technique now known as cohomological induction or the derived functor construction of representations. An important special case of this construction are the so-called $A_q(\lambda)$ representations which are cohomologically induced from one-dimensional characters. The paper Branching Problems of Zuckerman Derived Functor Modules by Toshiyuki Kobayashi provides a comprehensive survey of known results on the restrictions of the $A_q(\lambda)$ to symmetric subgroups, along with sketches of the most important ideas of the proofs.

Chiral Equivariant Cohomology of Spheres, by Bong H. Lian, Andrew R. Linshaw, and Bailin Song, is a survey of their work on the theory of chiral equivariant cohomology. This is a new topological invariant which is vertex algebra valued and contains the Borel-Cartan equivariant cohomology theory of a $G$-manifold as a substructure. The paper describes some of the general structural features of the new invariant—a quasi-conformal structure, equivariant homotopy invariance, and the values of this cohomology on homogeneous spaces—as well as a class of group actions on spheres having the same classical equivariant cohomology, but which can all be distinguished by the new invariant.
An irreducible admissible representation of a Lie group $G$ is determined by its global character, which is an invariant distribution. It is represented by a conjugation invariant function defined on a dense subset of the semisimple elements. In *Computing Global Characters* Jeffrey Adams gives an explicit algorithm for such characters, based on the Kazhdan-Lusztig-Vogan polynomials.

Part of Arthur’s conjectures predict the existence of certain stable virtual characters associated to nilpotent orbits. These conjectures are known in the case of an Archimedean field. In *Stable Combinations of Special Unipotent Representations*, Dan M. Barbasch and Peter E. Trapa study the space of stable virtual representations associated certain nilpotent orbits $O$, and relate this space to the *special piece* of $O$.

*Levi Components of Parabolic Subalgebras of Finitary Lie Algebras*, by Elizabeth Dan-Cohen and Ivan Penkov, turns to understanding certain “standard” subalgebras of the finitary Lie algebras including $sl(\infty)$, $so(\infty)$ and $sp(\infty)$. A lot is known about their Cartan, Borel and parabolic subalgebras. This paper goes much further by giving a description of subalgebras which can appear as the Levi component of a simple finitary Lie algebra, and a characterization of all parabolic subalgebras of which a given subalgebra is a Levi component.

In *On Extending the Langlands-Shahidi Method to Arithmetic Quotients of Loop Groups*, Howard Garland describes an important new approach to proving analytic properties of L-functions on certain finite dimensional groups by using the theory of Eisenstein series on infinite dimensional loop groups. This theory of Eisenstein series on loop groups for minimal parabolics has been developed by him in a series of previous papers. In this paper, he shows how the theory changes when one works with Eisenstein series associated to cusp forms on maximal parabolics.

If a group $K$ acts on a vector space $V$ then one is often interested in understanding the algebra of invariant polynomials $S(V^*)^K$ as explicitly as possible. In the paper *The Measurement of Quantum Entanglement and Enumeration of Graph Coverings*, motivated by questions in quantum computing, Michael W. Hero, Jeb F. Willenbring, and Lauren Kelly Williams consider the case of the group $K = U(n_1) \times \cdots \times U(n_r)$ acting in the usual manner on the tensor product $V = \mathbb{C}^{n_1} \otimes \cdots \otimes \mathbb{C}^{n_r}$, and they determine the invariant polynomials in the limiting case where all the $n_i \to \infty$.

Roger Howe’s theory of dual pairs typically refers to a commuting pair of subgroups of the symplectic group. Although Howe’s original formulation was in terms of Lie superalgebras, and such algebras have a wide variety of applications, their dual pairs have not received much attention. In *The Dual Pairs $(O(p,q), O\tilde{Sp}_{2,2})$ and Zuckerman Translation*, Dan Lu and Roger Howe study the representation theory of the dual pair consisting of the group $O(p,q)$ and the Lie superalgebra $O\tilde{Sp}_{2,2}$.

The paper *On the algebraic set of singular elements in a complex simple Lie algebra* by Bertram Kostant and Nolan Wallach studies a space of defining equations, denoted $M$, for the cone of singular elements of a semisimple Lie algebra $\mathfrak{g}$. The main result of the paper is the explicit decomposition of $M$ as a $\mathfrak{g}$-module.

In *An Explicit Embedding of Gravity and the Standard Model in $E_8$*, Garrett Lisi gives some preliminary steps in the construction of a unified theory of all
interactions based on the gauge group $E_8$. The kinematic framework he proposes accounts for one of the three observed generations of matter, but also include many particles that are not yet observed. His description includes an explicit embedding of the standard model and gravitational gauge groups into $E_8$, and the action of the corresponding Lie algebra generators on fermions.

Harmonic analysis on a connected reductive algebraic group $G$ is a special case of harmonic analysis on symmetric spaces: one writes $G = G \times G/G^\delta$ where $G^\delta$ is the fixed points of the involution interchanging the two factors. In *From Groups to Symmetric Spaces* George Lusztig studies various properties which are known in the group case, and to what extent they generalize to other symmetric spaces.

In *Study of Antiorbital Complexes* George Lusztig studies a problem on the support of the Fourier transform over a finite field. Special cases are related to cuspidal character sheaves and canonical bases.

A “classical” automorphic representation $V$ of a group $G$ corresponds to an imbedding $V \hookrightarrow L^2(G/\Gamma)$. Composing this with evaluation at identity, one obtains an automorphic distribution. The paper *Adelization of Automorphic Distributions and Mirabolic Eisenstein Series* by Stephen D. Miller and Wilfried Schmid analyzes the mirabolic Eisenstein series attached to a congruence subgroup of $GL(n, \mathbb{Z})$ and its associated automorphic distribution.

In an algebraic setting, Ivan Penkov and Vera Serganova propose a systematic approach to studying various categories of modules over the Lie algebras $sl(\infty)$, $o(\infty)$, and $sp(\infty)$. One of the main results of *Categories of Integrable $sl(\infty)$-, $o(\infty)$-, $sp(\infty)$-modules* states that an integrable module with finite dimensional weight spaces is semisimple. Another important result gives a description of the largest category of integrable modules which is closed under dualization and whose objects have finite Loewy lengths.

Macdonald polynomials are a far-reaching generalization of a number of important special functions in representation theory and combinatorics. The expansion coefficients of a product of two Macdonald polynomials may be regarded as generalized Littlewood-Richardson coefficients. The paper *Binomial Coefficients and Littlewood–Richardson Coefficients for Interpolation Polynomials and Macdonald Polynomials* by Siddhartha Sahi obtains explicit formulas for these coefficients by solving a more general problem involving the interpolation polynomials introduced by Knop and Sahi.

The paper *Restriction of some Representations of $U(p,q)$ to a Symmetric Subgroup* by Birgit Speh studies the restriction of derived functor modules of $U(p,q)$ to certain noncompact symmetric subgroups. It is shown that in various cases the decomposition is discrete with finite multiplicities.
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This volume contains the proceedings of the conference on Representation Theory and Mathematical Physics, in honor of Gregg Zuckerman’s 60th birthday, held October 24–27, 2009, at Yale University.

Lie groups and their representations play a fundamental role in mathematics, in particular because of connections to geometry, topology, number theory, physics, combinatorics, and many other areas. Representation theory is one of the cornerstones of the Langlands program in number theory, dating to the 1970s. Zuckerman’s work on derived functors, the translation principle, and coherent continuation lie at the heart of the modern theory of representations of Lie groups. One of the major unsolved problems in representation theory is that of the unitary dual. The fact that there is, in principle, a finite algorithm for computing the unitary dual relies heavily on Zuckerman’s work.

In recent years there has been a fruitful interplay between mathematics and physics, in geometric representation theory, string theory, and other areas. New developments on chiral algebras, representation theory of affine Kac-Moody algebras, and the geometric Langlands correspondence are some of the focal points of this volume. Recent developments in the geometric Langlands program point to exciting connections between certain automorphic representations and dual fibrations in geometric mirror symmetry.