LECTURE 5 EXCERCISE SOLUTIONS

Problem. 1: With a truth table, verify the tautologies for Theorems E, C, O.

Solution. Note, the problem is to verify the **tautologies** in the proofs of the theorems, not the theorems themselves. They are tautologies, so you know the final column should be all truths. At a high level: For Theorem E, note that $P \iff Q$ is equivalent to $P \implies Q \bigwedge Q \implies P$. Letting the first be A and the second be B, you're essentially asked to verify that $(A \bigwedge B) \implies A$. So you should see why this is true. For Theorem C, at a high level, note that $Q \implies (P \bigwedge Q)$ is true if Q is false, and if Q is true, it all depends on P. For Theorem O, if Q is true, it doesn't matter what P is, and if Q is false, $P \bigwedge Q$ is equivalent to P.

Common Problems. People are mostly good here. Mind your p's and q's, if you will. A handful of people missed an entry or two in their truth tables, but this seemed to be a result of forgetting that $F \implies F$ is true, and $T \implies F$ is false, etc. Some people mix up their 'and's and 'or's. Also, a handful of people did the wrong problem. The problem asks you to verify the relevant tautologies for each theorem - not the theorem itself. Also, given that you are told they are tautologies, it's relatively easy to figure out what the final column of your table should be. The problem lies in getting there, so if you don't show enough of your computation to convince me, that's not enough. Always do enough to convince me you know what you are doing - and hopefully no more. As an added bonus, if you make a mistake (and this you should take as general advice), often I can see your mistake and give you partial credit, rather than nothing.

Problem. 2: Show that $(P \bigwedge Q) \bigwedge R$ is equivalent to $P \bigwedge (Q \bigwedge R)$ using a truth table.

Solution. Note, if any one of P, Q, R are false, both expressions are false. And if all three are true, both expressions are true.

Common Problems. Again, most of the problems I saw were people forgetting the results of various simple propositions. But this was a little more worrying, as a handful of people all made the identical mistake, stating things like $F \bigwedge F$ was true. Don't do that...

Problem. 3: Consider the statements,$\sim P \implies K, M \implies P, C \implies \sim S, \sim S \implies C, C \bigvee L, S \bigvee P$. Using those and $\sim L$ as true, conclude P.

Solution. Noting $\sim L$ and $C \bigvee L$ are true, by Theorem O, C is true. Because C is true and $C \implies \sim S$ is true, $\sim S$ is true. Since $\sim S$ is true and $S \bigvee P$ is true, by Theorem O, P is true.

Common Problems. A big problem here was people not justifying themselves. For instance, people might list which of the above statements they needed, and say that they implied P. That is insufficient for credit, since you didn't state -how- they implied P, such as citing theorems, or in some way conveying your logic. Some people would use hypothesis that you had no information about. The fact that $M \implies P$ doesn't help you, unless you know something about M. Remember, $F \implies F$ is true. Some people would take as true things you were not given explicitly, such as $\sim S$. Always make your logic clear. Start with things you know. And if those things are not given explicitly (such as $\sim S$, though it is ultimately true), justify why you know them.

Problem. 4: This exercise was not graded.