Math 135, Section C7 - Review problems for Exam #1 - June 14, 2010

#1 Find all $x$ such that $|x - 3| < \frac{7}{2}$ and express your answer in interval notation.

#2 Write an equation for a straight line:
   (a) which passes through the point $(1, -2)$ and has slope 3;
   (b) which passes through the points $(3, 5)$ and $(5, -8)$
   (c) which passes through the point $(-4, 1)$ and is parallel to the straight line with equation $y = -2x + 7$;
   (d) which passes through the point $(-1, 1)$ and is perpendicular to the line through $(7, 4)$ and $(2, 2)$.

#3 Write an equation of the circle with center $(3, 2)$ and radius 5.

#4 (a) The graph of the equation $x^2 + y^2 - 2x + 4y - 4 = 0$ is a circle. What are the center and radius of this circle?
   (b) The graph of the equation $7x - 5y + 23 = 0$ is a straight line. What is its slope? If the point $(a, 2a)$ is on this line, what is $a$?

#5 Suppose that $f(x) = 2x - 1$ if $x < 1, f(1) = a$ and $f(x) = 3x + b$ if $x > 1$. Suppose further that $f(x)$ is continuous at $x = 1$. What are $a$ and $b$? Explain why using the definition of continuity.

#6 Find each of the following limits or state that the limit does not exist:
   (a) $\lim_{x \to 2} (x^2 + \frac{x}{x-1})$
   (b) $\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$
   (c) $\lim_{x \to 2^+} \frac{x - 2}{|x - 2|}$
   (d) $\lim_{x \to 2^-} \frac{x - 2}{|x - 2|}$
   (e) $\lim_{x \to 2} \frac{x - 2}{|x - 2|}$
   (f) $\lim_{x \to 2^+} \frac{1}{|x - 2|}$
   (g) $\lim_{x \to 4} \frac{x - 4}{\sqrt{x - 2}}$

#7 Use the definition of derivative to find
   (a) $f'(x)$ if $f(x) = x^2 + x + 1$
#8 In each part, find \( f'(x) \) by any method:

(a) \( f(x) = x^3 + 2x^2 - x + 3 \)

(b) \( f(x) = x\sqrt{x} + 3\frac{1}{\sqrt{x}} \)

(c) \( f(x) = \sin(2x + 3) \)

(d) \( e^{(2x+3)} \)

(e) \( f(x) = e^{\sin(x)} \)

(f) \( f(x) = \frac{\sin(x)}{e^{2x+3}} \)

(g) \( f(x) = \sqrt{\frac{x^2+1}{x^2+2}} \)

(h) \( f(x) = (x^3 + 2x)^{17} \)

(i) \( f(x) = \ln(\sin(2x + 3)) \)

(j) \( f(x) = x^2\sin(e^{2x} + 3) \)

#9 A straight east-west road goes through the town of Bend. Suppose that at time \( t \) (in hours), where \( 0 \leq t \leq 10 \), a car is \( 20 + 8t - t^2 \) miles east of Bend.

(a) What is the velocity of the car at time \( t \)?

(b) What is the speed of the car at time \( t \)?

(c) What is the acceleration of the car at time \( t \)?

(d) What is the total distance traveled by the car between \( t = 1 \) and \( t = 7 \)?

#10 Suppose \( f(x) \) and \( g(x) \) are two functions which are defined for all real numbers. Suppose that

\[
\begin{align*}
    f(-2) &= 1, f(-1) = 0, f(0) = 2, f(1) = 1, f(2) = -1, \\
    g(-2) &= -2, g(-1) = 1, g(0) = 0, g(1) = -2, g(2) = 2, \\
    f'(-2) &= 0, f'(-1) = 3, f'(0) = -3, f'(1) = 2, f'(2) = -1, \\
    g'(-2) &= 2, g'(-1) = -1, g'(0) = 2, g'(1) = -2, \text{ and } g'(2) = 3.
\end{align*}
\]

Let \( h(x) = f(g(x)) \) and \( p(x) = g(f(x)) \). Find:

(a) \( h(2) \)
(b) $h'(2)$
(c) $p(2)$
(d) $p'(2)$

#11 Let

$$f(x) = 1 - x^2, \text{ if } x < 2;$$

and

$$f(x) = ax + b, \text{ if } x \geq 2.$$  

Suppose $f(x)$ is differentiable at $x = 2$. What are $a$ and $b$? Why?

#12

Let

$$g(x) = x + 3, \text{ if } x \leq 1;$$

and

$$g(x) = x^2 + 3, \text{ if } x > 1.$$  

Is $g(x)$ continuous at $x = 1$? Is $g(x)$ differentiable at $x = 1$? Explain your answers using the definitions.

#13 Show that

$$t^2 + 1 = \frac{10}{3t^2 + 2}$$

for some $t$ in the interval $[-2, 2]$.

#14 A block of ice in the shape of a cube originally has volume 1,000 cubic centimeters. It is melting in such a way that it maintains its cubical shape at all times and that the length of each of its edges is decreasing at the rate of 1 centimeter per hour. At what rate is its surface area decreasing at the time its volume is 27 cubic centimeters?

#15 Find an equation for the tangent line to the graph of $y = e^{x^2 + 3}$ at the point where $x = 1$. 