Practice questions for exam #1

I will work these problems at a review session on Sunday, 4/3, 3-5 PM in ARC-309 and again at a review session on Tuesday, 4/5, 7-9 PM in ARC-309. (ARC-309 is in the Mathematics and Science Learning Center on the third floor of ARC.) Remember that the exam is in class on Thursday, 4/7. Solutions to these problems will be posted after the 4/3 review session.

#1 Consider the linear programming problem

Maximize: $c^T x$

Subject to:

$Ax \leq b$

$x \geq 0$.

This problem is in standard form.

(a) State the dual problem.

(b) Show that if $x$ is any feasible solution to the primal problem and $w$ is any feasible solution to the dual problem then $c^T x \leq b^T w$.

(c) Show that if the primal problem is unbounded, then the dual problem is infeasible.

(d) Show that the dual of the dual of the given problem is the primal problem.

#2 Consider the linear programming problem

Maximize: $r^T x$

Subject to:

$Ax = s$

$x \geq 0$

This problem is in canonical form. Find the dual of this problem, by writing the primal problem in standard form and using your answer to #1. Explain
why the dual involves unrestricted variables.

#3 Find the dual of the linear programming problem:

Minimize: \(-3x_1 + 2x_2 + x_4\)
Subject to:
\[\begin{align*}
2x_1 + x_2 + x_3 + 2x_4 &\geq 7 \\
x_2 + 3x_4 &= 5 \\
x_1, x_2 \geq 0, x_3 \leq 0, x_4 \text{ unrestricted.}
\end{align*}\]

#4 Use the revised simplex method to solve the linear programming problem

Maximize: \(2x_1 + x_2 + 3x_3 + x_6 + 2x_7 + 3x_8\)
Subject to:
\[\begin{align*}
2x_1 + x_2 + x_4 + 3x_5 + x_7 &\leq 24 \\
x_1 + 3x_3 + x_4 + x_5 + 2x_6 + 3x_8 &\leq 30 \\
5x_1 + 3x_2 + 3x_4 + 2x_5 + x_7 + 5x_8 &\leq 18 \\
3x_1 + 2x_2 + x_3 + x_6 + 3x_8 &\leq 20 \\
x_1, ..., x_8 &\geq 0.
\end{align*}\]

Give the current \(B^{-1}\) and the current list of basic variables at each step.

#5 Consider the linear programming problem:

Maximize: \(4x_1 + 3x_2 + 6x_3\)
Subject to:
\[\begin{align*}
3x_1 - 4x_2 - 6x_3 &\leq 18 \\
-2x_1 - x_2 + 2x_3 &\leq 12 \\
x_1 + 3x_2 + 2x_3 &\leq 1 \\
x_1, x_2, x_3 &\geq 0.
\end{align*}\]

The optimal solution to this problem is \(z = 4\) at the point \(x_1 = 1, x_2 = 0, x_3 = 0\) and the final tableau for the simplex method is:
(a) State the dual problem and find its optimal solution
(b) Find all values of $\Delta c_2$ such that the solution above remains optimal.
(c) Find all values of $\Delta c_5$ such that the solution above remains optimal.
(d) Find the optimal solution of the problem obtained by changing $c_6$ to 3.
(e) Suppose the final tableau is obtained from the initial tableau by multiplying by $B^{-1}$. Find $B^{-1}$.
(f) Find the optimal solution to the problem obtained by changing the constant term in the third constraint ($b_3$) from 1 to 5.
(g) Find the optimal solution to the problem obtained by changing the constant term in the third constraint ($b_3$) from 1 to 7.
(h) A further constraint $x_2 + x_3 \geq 1$ is added to the original problem. Use the dual simplex method to find an optimal solution to this new problem (if one exists).
(i) A different further constraint $2x_1 + x_2 \leq 1$ is added to the original problem (not to the modified problem in (h)). Use the dual simplex method to find an optimal solution to this new problem (if one exists).

#6 Find an optimal solution to the following pure integer programming problem.

Maximize: $x_1 + 3x_2$

Subject to:

\[
\begin{align*}
  x_1 - 2x_2 & \geq 0 \\
  x_1 + 2x_2 & \leq 42 \\
  x_1, x_2 & \geq 0, \ x_1, x_2 \text{ integers}
\end{align*}
\]
#7 Find an optimal solution to the following pure integer programming problem.

Maximize: $x_1 + 2x_2 + x_3 + x_4$

Subject to:

- $2x_1 + x_2 + 3x_3 + x_4 \leq 8$
- $2x_1 + 3x_2 + 4x_4 \leq 12$
- $3x_1 + x_2 + 2x_3 \leq 18$
- $x_1, x_2, x_3, x_4 \geq 0$, $x_1, x_2, x_3, x_4$ integers