(20) 1. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\lim_{x\to 3} \dfrac{x^2-x-6}{x-3}$

b) $\lim_{x\to -2^-} \dfrac{x^2-x-6}{|x+2|}$

c) $\lim_{x\to\infty} \dfrac{4x-7}{e^{3x}}$

d) $\lim_{x\to 0} \dfrac{\sin(3x)}{\cos(2x)}$

(12) 2. Suppose $f(x) = \frac{1}{x^2}$. Use the **definition of derivative** to find $f'(x)$.

(22) 3. Compute the derivatives with respect to x of the following functions.
Please do not simplify the answers.

a) $\cos\left(x^4+3\right)$

b) $\left(e^{7x}+3x^4\right)\left(\sqrt{x+5}+\dfrac{2}{x^3}\right)$

c) $\dfrac{x^3+2}{5\ln x}$

d) $\displaystyle\int_{-3}^{x} \sin\left(t^3\right)\,dt$

(8) 4. Suppose $f(x) = \left(x^2+9\right)^{200} - \left(17-x^3\right)^{301}$.
a) Compute $f'(x)$. Explain why the following statement is correct: if $x > 0$, then $f'(x) > 0$.
b) Use calculus to explain why $f(76) > f(23)$. You must quote a specific result from this course and explain its relevance. Your answer to a) may be useful here.

(16) 5. A rectangular box with a square base is to be made from two different materials. The material for the top and four sides costs \$1 per square foot, while the material for the bottom costs \$2 per square foot. If you can spend \$196 on materials, what dimensions will maximize the volume of the box?

(10) 6. Find equations for all horizontal and vertical asymptotes of $f(x) = \frac{3+2e^{2x}}{5-7e^{2x}}$.

(10) 7. a) Give an example of a function which is not continuous. Explain why your example is not continuous.
b) Give an example of a function which is not differentiable. Explain why your example is not differentiable.

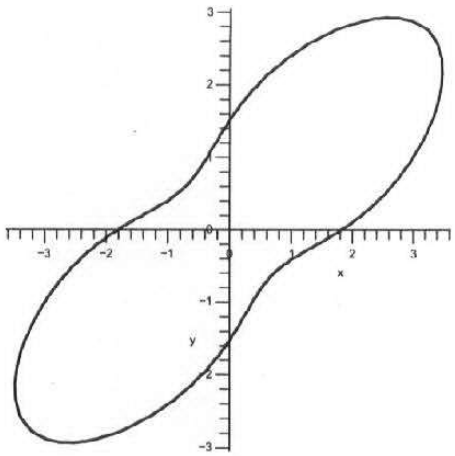
(12) 8. Suppose that y is implicitly defined as a function of x by the equation $x^4 - 24xy + 2y^4 = 11$.

a) Find $\dfrac{dy}{dx}$ in terms of y and x.

b) Find an equation for the line tangent to the graph at the point $P = (3, 1)$ which is on the graph.

Note There are some **Possibly useful numbers** on the formula sheet.

c) The program `Maple` gives the image shown to the right when asked to graph the equation. Sketch the tangent line found in b) on the image.



(18) 9. Suppose that $f'(x)$, the *derivative* of $f(x)$, is given by this formula:
$f'(x) = (x+2)x^2(x-1)^3$.

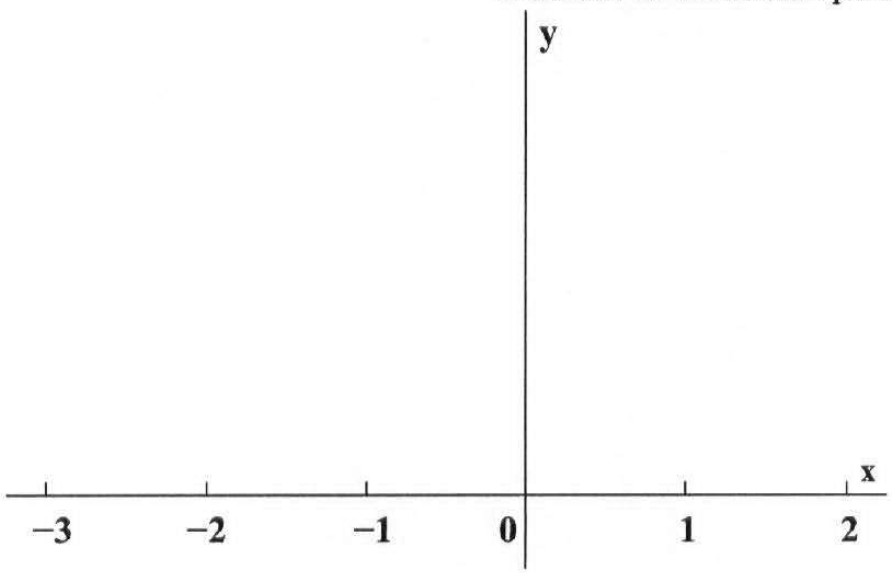
Note Read the formula carefully. Please do not try to find a formula for $f(x)$: this is not requested and will not help you answer any part of the problem.

a) What are the critical numbers of $f(x)$? For each critical number, explain why the associated critical point is a relative minimum, a relative maximum, or neither. Briefly support your answers using calculus.

b) Sketch a graph of $y = f(x)$ showing the features found in a) on the axes given. The graph should be as simple as possible. Label each relative maximum with **M** and label each relative minimum with **m**.

c) How many inflection points does your graph of $y = f(x)$ have? Label each inflection point with **I** on the graph drawn. Please do not compute $f''(x)$: use the graph.

Number of inflection points _____



(12) 10. A spherical hot air balloon is being filled with air at the rate of 200 cubic feet per minute. At what rate is the radius of the balloon increasing when the balloon has 1000 cubic feet of air in it?

(12) 11. Suppose $f(x) = 3x^4 - 8x^3 - 18x^2 + 2$.
a) Find the exact maximum and minimum values of $f(x)$ for x in the interval $[-2, 2]$. Briefly explain your conclusions using calculus.
Note There are some **Possibly useful numbers** on the formula sheet.
b) If $[-2, 2]$ is the domain of the function $f(x)$, describe the range of $f(x)$ precisely. You may use your results in a) here. You must quote a specific result from this course and explain its relevance.

(8) 12. Compute the value of the Riemann sum for the function 3^x on the interval $[-1, 3]$ using the partition $\{-1, 0, 2, 3\}$ and taking as the sample points the right-hand endpoints of each subinterval.

(13) 13. Find the following indefinite integrals.

a) $\int \left(5\sqrt{x} - \frac{3}{x^2}\right) dx$

b) $\int e^{2x+x^2}(1+x)\, dx$

c) $\int \Big(\cos(x+3) + \sin(2x)\Big)\, dx$

(10) 1. Evaluate the indicated limits exactly. Give brief evidence supporting your answers which is not based on a calculator graph or calculator computations.

a) $\lim_{x\to\infty} \dfrac{2x^2-5}{3x^2+1}$.

b) $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$.

(12) 2. Find the equations of all vertical and horizontal asymptotes of the function

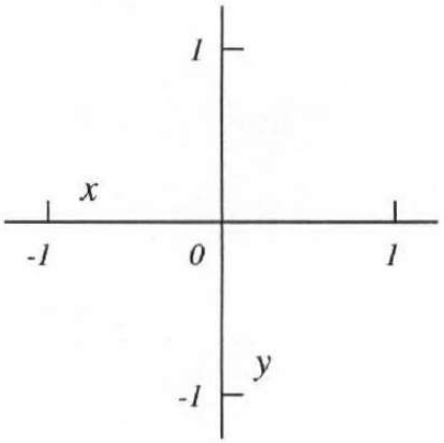
$$f(x) = \frac{3e^x+5}{7e^x-2}.$$

Computations with exp and log should be simplified as much as possible. A numerical approximation like 1.40135 is **not** acceptable.

(18) 3. a) Write the definition of derivative as a limit and *use this definition* to find the derivative of $F(x) = x - x^2$.

b) Use your answer to a) to find the equation of a line tangent to $y = x - x^2$ at the point where $x = 0$.

c) Sketch $y = x - x^2$ and the line found in b) on the axes given.



(14) 4. Find $\dfrac{dy}{dx}$ for each of the following:

a) $y = \dfrac{2x^2+5}{5x^3+1}$ b) $y = (4x+3)\sqrt{x^3+7}$ c) $xy^3 = \cos(7x+5y)$

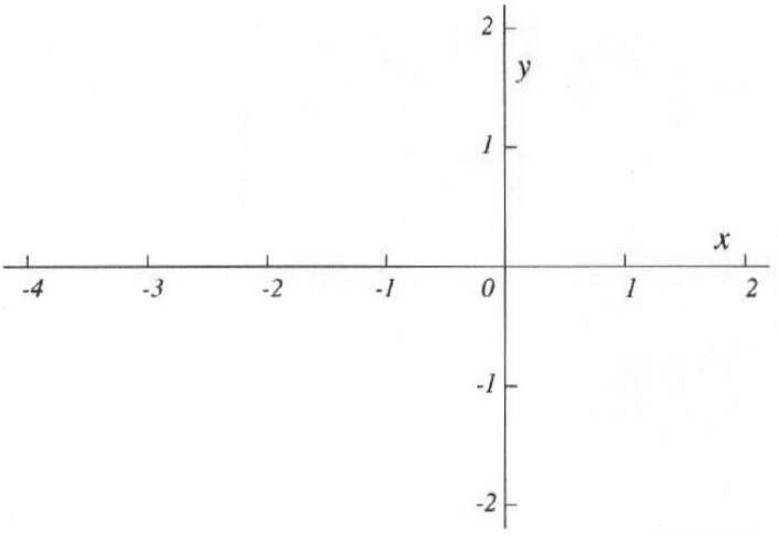
(16) 5. In this problem

$$W(x) = \begin{cases} x+3 & \text{if } x \le -2 \\ \frac{1}{2}x^2 + A & \text{if } -2 < x \end{cases}$$

where A is a constant to be determined in part a).

a) Find A so that the function is continuous for all values of x.

b) Sketch a graph of $y = W(x)$ for $-4 \le x \le 2$ using the value of A found in a) on the axes given.



c) Is $W(x)$ differentiable at $x = -2$ using the value of A you have found in part a)? Explain your answer briefly.

(18) 6. Suppose that $N(x) = 5x^3 - 3x^5$.

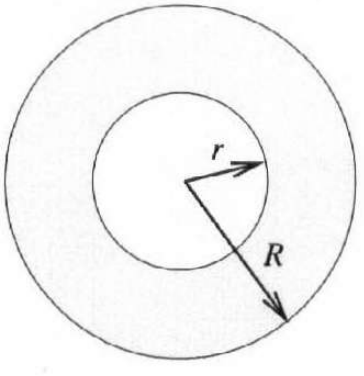
a) Compute $N'(x)$ and $N''(x)$. Where are each of these functions equal to 0?

b) Find all relative maximum and minimum values of $N(x)$. Briefly explain your answers using calculus.

c) Find all points of inflection of $N(x)$. Briefly explain your answers using calculus.

(12) 7. Two circles have the same center. The inner circle has radius r which is increasing at the rate of 3 inches per second. The outer circle has radius R which is increasing at the rate of 2 inches per second. Suppose A is the area of the region *between* the circles.

At a certain time, r is 7 inches and R is 10 inches. What is A at that time? How fast is A changing at that time? Is A increasing or decreasing at that time?



(16) 8. A box with an open top is to be made from rectangular sheet of cardboard 5 inches by 8 inches by cutting equal squares out of the four corners and bending up the resulting four flaps to make the sides of the box. Use calculus to find the largest volume of the box.

Be sure to explain briefly why your answer gives a maximum.