#1 Use mathematical induction to prove that 8 divides $5^{2n} - 1$ for every integer $n \geq 1$.

#2 Use the well ordering principle to show that if $a, b$ are natural numbers then there exist integers $q, r \geq 0$ such that $a = qb + r$ and $0 \leq r < b$.

#3 Suppose $\overline{A} = 33, \overline{B} = 17$, and $\overline{A \cap B} = 12$. Find $\overline{A \cup B}$.

#4 Suppose $\overline{A} = 11$.
   (a) Find $\overline{\mathcal{P}(A)}$.
   (b) How many subsets $B \subseteq A$ satisfy $\overline{B} = 4$?

#5 (a) State the definition of a relation from $A$ to $B$.
   (b) Suppose $\overline{A} = 7, \overline{B} = 5$. How many relations from $A$ to $B$ are there?

#6 Let $A = \{1, 2, 3, 4, 5\}$. For each of the following relations from $A$ to $A$ state whether or not it is an equivalence relation. If it is an equivalence relation give the corresponding partition of $A$.
   (a) $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 4), (5, 5)\}$.
   (b) $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 4), (4, 5), (5, 5)\}$.

#7 (a) State the reflexive, symmetric, anti-symmetric, and transitive properties of a relation.
   (b) State the definition of an equivalence relation, of a partial order, of a total order, and of a partition.

#8 For $0 \leq n \leq 6$, let $J_n = \{k \in \mathbb{Z} | 7 \text{ divides } k - n\}$. Show that $\{J_0, ..., J_6\}$ is a partition of the integers and describe the corresponding equivalence relation.

#9 (a) Define a relation $R$ on the integers by $aRb$ if and only if either $a = b$ or $a + 2 < b$. Is $R$ a partial order? Is it a total order?
   (b) Define a relation $S$ on the integers by $aSb$ if and only if $a$ and $b$ have the same parity (i.e., both are even or both are odd) and $a \leq b$. Is $S$ a partial order? Is it a total order?

#10 State the definition of a function $f$ from a set $A$ to a set $B$. State the definition of the domain, codomain, and range of $f$.

#11 Let $A = \{1, 2, 3, 4\}, B = \{x, y, z\}$. Let $f$ be the function from $A$ to $B$ defined by
\[ f(1) = z, f(2) = x, f(3) = y, f(4) = z \] and \( g \) be the function from \( B \) to \( A \) defined by \( g(x) = 4, g(y) = 3, g(z) = 2 \). Find \( f \circ g \) and \( g \circ f \). Is either of the functions \( f \circ g, g \circ f \) one-to-one? Onto?

#12 Suppose \( f \) is a function from \( A \) to \( C \) an \( g \) is a function from \( B \) to \( C \). When will \( f \cup g \) be a function? Why?

#13 State the definition of: \( \lim_{n \to \infty} x_n = L \).

#4 (a) Show that \( \lim_{n \to \infty} \frac{n+1}{1-2n} = -\frac{1}{2} \).

(b) Show that \( \lim_{n \to \infty} \frac{2^n}{3^n} = 0 \).

(c) Show that the sequence given by \( x_n = (-1)^n(1 - \frac{1}{n}) \) diverges.