Rational generating functions and compositions
by Bruce Sagan (joint work with Anders Björner)

A composition of the nonnegative integer \( n \) is a way of writing \( n \) as an ordered sum. So the compositions of 3 are \( 1 + 1 + 1 \), \( 1 + 2 \), \( 2 + 1 \), and 3 itself. It is well-known and easy to prove that if \( c_n \) is the number of compositions of \( n \) then \( c_n = 2^{n-1} \) for \( n \geq 1 \) and \( c_0 = 1 \). Equivalently, we have the generating function

\[
\sum_{n \geq 0} c_n x^n = \frac{1 - x}{1 - 2x}
\]

which is a rational function. We show that this is a special case of a more general family of rational functions associated with compositions. Our techniques include the use of formal languages. Surprisingly, identities from the theory of hypergeometric series are needed to do some of the computations.