Presentation Outline

Introducing Proof Maps

Pedagogical Uses

Classroom Experience

Next steps
Motivation

Clarify the structure of proofs by breaking it into discrete logical steps using diagrams.

- “Hidden” arguments lurking in our language
- Proof methods versus logical arguments
- Like text messages: *where is the emphasis?*
- Cases
- Contradiction
- Logical Flow
- What is being proved?
Searching for structure

Much of the work of working through a proof is determining its structure

- True for both articles and textbooks

Stripping away the language helps this process

Arguments are typically non-linear

- Diagrams are appropriate
Definition

A proof map is a diagrammatic representation of a proof

- Main steps of the proof are written in bubbles
- Lemmas used are written in boxes
- Arrows are drawn in between to denote implication
Basic Format

Assumptions → Arguments → Conclusions
Introducing Proof Maps

Basic example

### Multiplicative inverses are unique

- $y$ is an inverse of $x$
- $z$ is an inverse of $x$
- $yx = xy = 1$
- $zz = zz = 1$
- $y(xz) = (yx)z$
- $y \cdot 1 = 1 \cdot z$
- $y = z$
Additional Notation

Additional notation to accelerate proof interpretation

- Hearts next to the most important items
- Lightbulbs next to the clever tricks
- Dotted bubbles around assumptions made towards a contradiction
- Lightning bolts next to the contradictions
Another example

Proof by Contradiction

\[ P := \{p_1, \ldots, p_n\} \text{ is the set of prime numbers.} \]

\[ N := p_1p_2\cdots p_n + 1 \]

\[ p_i \nmid N \text{ for all } i \in \{1, \ldots, n\} \]

\[ q = p_i \text{ for some } i \in \{1, \ldots, n\} \]

\[ \exists q \text{ prime, } q \mid N \]
Presentation Outline

Introducing Proof Maps

Pedagogical Uses

Classroom Experience

Next steps
Less language, more comprehension

- A big block of text can be overwhelming
- Subtle linguistic choices convey messages which the students may miss
- Easier to remember
- Trains students to analyze proofs the way mathematicians do
Reinforcing visual learning

- Proofs are often non-linear - proof maps make this more transparent
- Visual learners may more easily remember a proof map
- Makes logical gaps more obvious
Presentation Outline

Introducing Proof Maps

Pedagogical Uses

Classroom Experience

Next steps
Implemented proof map techniques in an introduction to math reasoning course

- Workshop 30 minutes a week in addition to homework problems

Messy data

- Unable to deduce concrete results
Helpful techniques

Proof maps may be useful in creating a workshop for math reasoning courses

▶ Two truths and a lie a good icebreaker
▶ Creates a concrete, potentially collaborative activity in connection with a proof
▶ Fill-in-the-blank and connect-the-bubbles spurred engagement
▶ Matching tended to be “meta-reasoned”
▶ Proof mapping a part of a proof may be appropriate early on
Presentation Outline

Introducing Proof Maps

Pedagogical Uses

Classroom Experience

Next steps
Going forward

Experimenting with a slightly higher level course

- Considering introducing them into an into to analysis course
Thank you!

Alejandro Ginory aginory@math.rutgers.edu

Rebecca Coulson rcoulson@math.rutgers.edu