Visualizing Mathematical Reasoning: A Diagrammatic Approach

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Presentation Outline

Motivation

Clarify the structure of proofs by breaking it into discrete logical steps using diagrams.

- "Hidden" arguments lurking in our language
- Proof methods versus logical arguments
- Like text messages: *where is the emphasis?*
- Cases
- Contradiction
- Logical Flow
- What is being proved?

Searching for structure

Much of the work of working through a proof is determining its structure

- True for both articles and textbooks

Stripping away the language helps this process
Arguments are typically non-linear

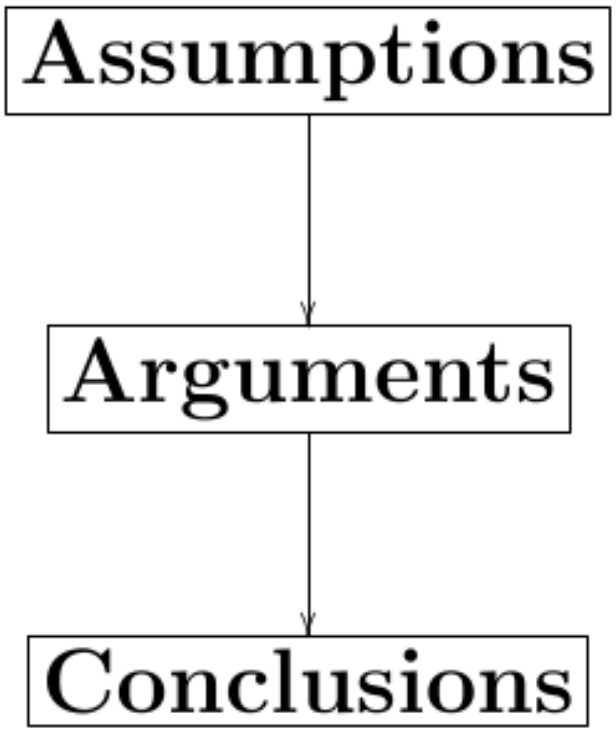
- Diagrams are appropriate

Definition

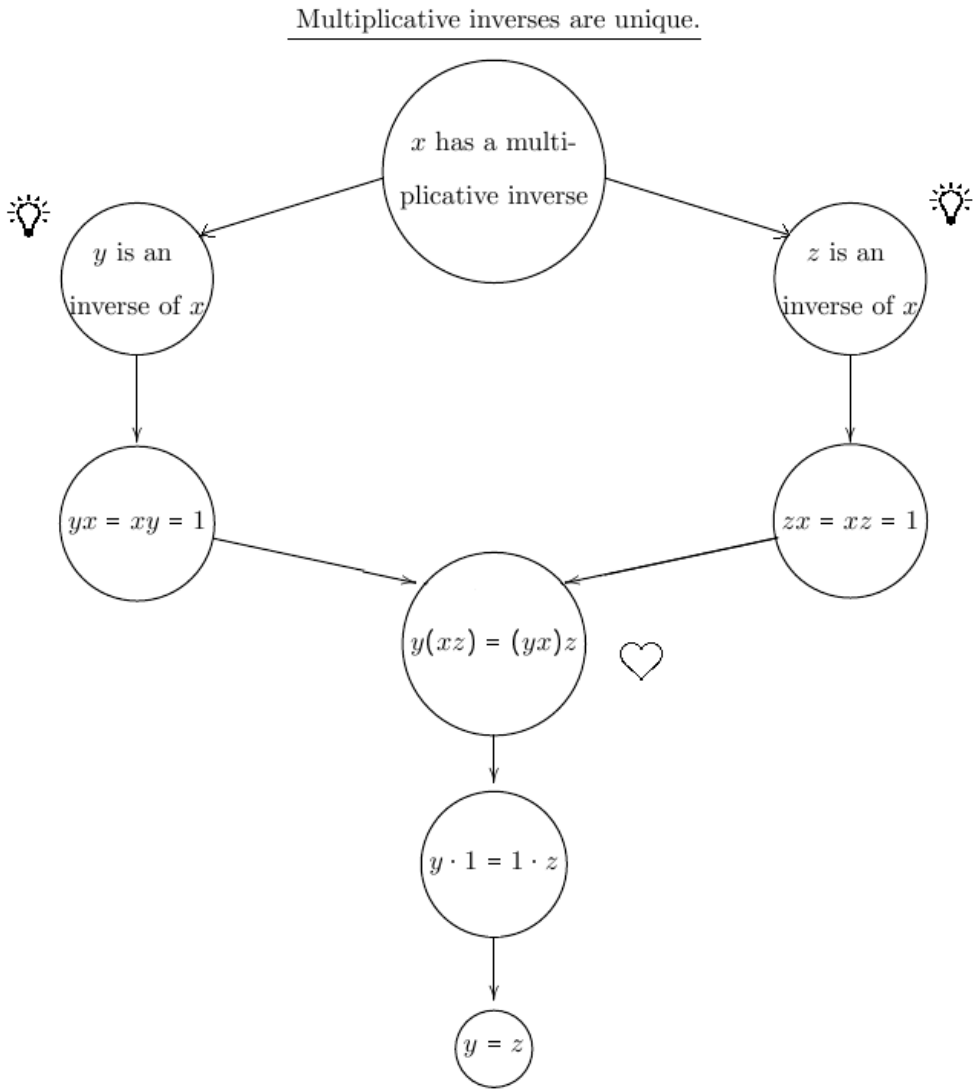
A proof map is a diagrammatic representation of a proof

- Main steps of the proof are written in bubbles
- Lemmas used are written in boxes
- Arrows are drawn in between to denote implication

Basic Format



Basic example

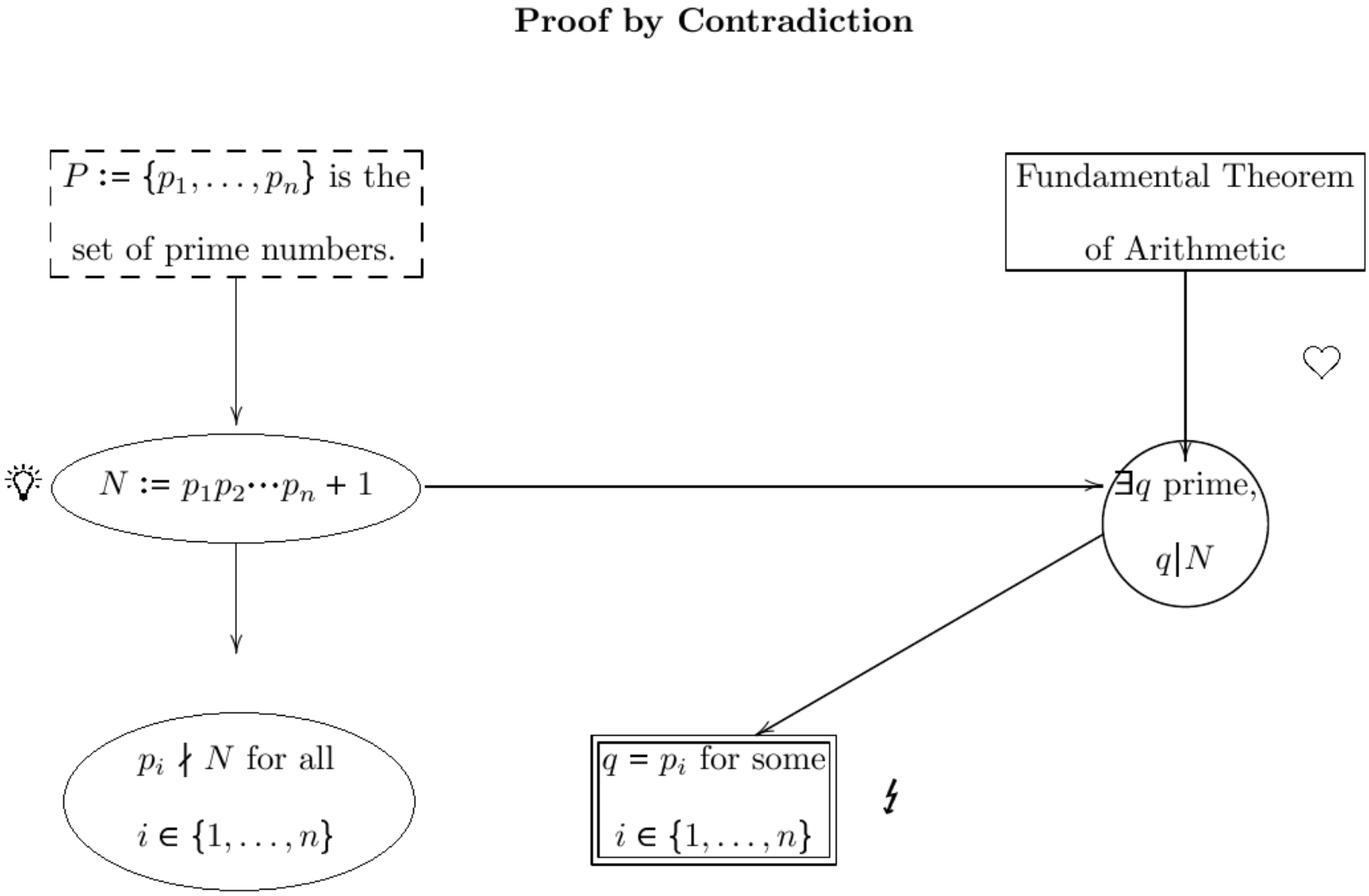


Additional Notation

Additional notation to accelerate proof interpretation

- Hearts next to the most important items
- Lightbulbs next to the clever tricks
- Dotted bubbles around assumptions made towards a contradiction
- Lightning bolts next to the contradictions

Another example



Presentation Outline

Less language, more comprehension

- A big block of text can be overwhelming
- Subtle linguistic choices convey messages which the students may miss
- Easier to remember
- Trains students to analyze proofs the way mathematicians do

Reinforcing visual learning

- Proofs are often non-linear - proof maps make this more transparent
- Visual learners may more easily remember a proof map
- Makes logical gaps more obvious

Presentation Outline

Background Info

Implemented proof map techniques in an introduction to math reasoning course

- Workshop 30 minutes a week in addition to homework problems

Messy data

- Unable to deduce concrete results

Helpful techniques

Proof maps may be useful in creating a workshop for math reasoning courses

- Two truths and a lie a good icebreaker
- Creates a concrete, potentially collaborative activity in connection with a proof
- Fill-in-the-blank and connect-the-bubbles spurred engagement
- Matching tended to be "meta-reasoned"
- Proof mapping a part of a proof may be appropriate early on

Presentation Outline

Going forward

Experimenting with a slightly higher level course

- Considering introducing them into an into to analysis course

Thank you!

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