Math 250 – Fall 2013
Quiz 6 - October 23, 2013 Name: _____

This quiz requires some computational work. You have 17 minutes.

1. (*7 points*) Consider the following matrix

$$C = \begin{pmatrix} 12 & 1 & 5.3 & 0 & 17 \\ 0 & 1 & 0 & 0 & 1.4 \\ 0 & 1 & 2 & 0 & -5 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{which has reduced row echelon form} \quad rref(C) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

(a) Is there a vector $\vec{b}$ in $\mathbb{R}^4$ such that the equation $C\vec{x} = \vec{b}$ is inconsistent? Explain.

(b) Is there a vector $\vec{x}$ in $\mathbb{R}^5$ other than $\vec{x} = \vec{0}$ such that $C\vec{x} = \vec{0}$? Explain.

(c) Let S be the solution set to $C\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$. Is S a subspace? Explain. [Hint: Don't figure out what that solution set is! Just start checking if it satisfies the properties of a subspace.]

2. (*6 points*) Suppose $T : \mathbb{R}^3 \to \mathbb{R}^4$ is a linear transformation such that

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -x_2 + 2x_3 \\ x_1 + x_3 \\ 6x_1 + x_2 \\ 3x_2 \end{bmatrix}.$$

(a) What is the standard matrix of T?

(b) Find a set of vectors that spans the image of T. [Hint: remember that the image of T is the same as the column space of its standard matrix.]

3. (*7 points*) Let $A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 6 \end{pmatrix}$ and $T_A(\vec{x}) = A\vec{x}$.

(a) Find the solution set to $T_A(\vec{x}) = \vec{0}$. Write your answer in vector form.

(b) Find the solution set to $T_A(\vec{x}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Write your answer in vector form.

(c) Find the solution set to $T_A(\vec{x}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Write your answer in vector form.

(d) Find the solution set to $T_A(\vec{x}) = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$. Write your answer in vector form.