Math 151 - Section C1
Summer 2014
Midterm 1
June 12, 2014
Time Limit: 80 Minutes

Name (Print): ____________________

Instructor: Pat Devlin

This exam has 13 pages (including this cover page and formula sheet) and 9 problems. Check to see if any pages are missing. Put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the place without a clear ordering is a pain the butt to read.
- **Mysterious or unsupported answers rarely receive credit**. Some answers require no work. However, the majority require at least some work or explanation or both.
- **When in doubt, show work and explanations**. But no question requires all that much explanation—don't waste your time writing a book!
- **If you get stuck** then move to another problem. Don't erase failed attempts, just put a line through them (that way you can get partial credit for attempts).
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	25	
2	20	
3	25	
4	25	
5	25	
6	25	
7	25	
8	30	
9	0	
Total:	200	

1. (25 points) At a Fourth of July parade, one of the large spherical balloons gets a hole in it, and air starts flowing out at a rate of 300 ft^3/min. At what speed is the radius of the balloon changing when the radius is 5 feet?

Good morning! I think the website xkcd.com is funny. Here are some comics for your amusement. No problems on this page. Just comics. Anyhow, good luck with the test!

Figure 1: Image from xkcd (626)

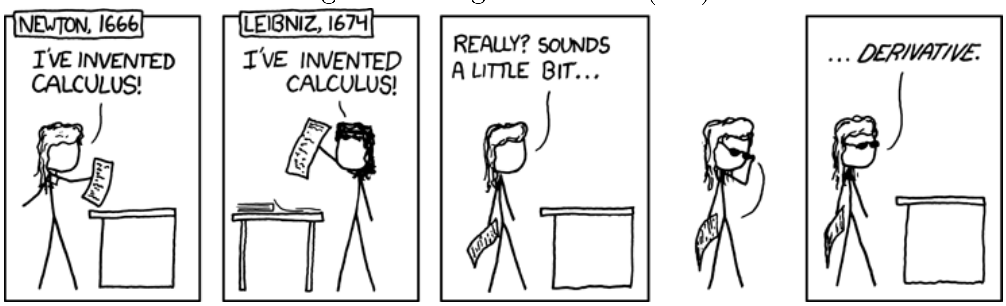
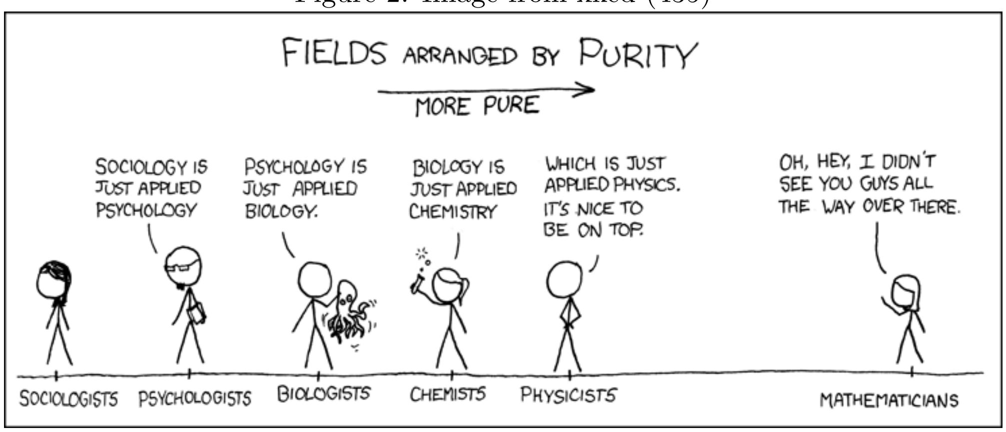


Figure 2: Image from xkcd (435)



2. (20 points) Compute the following limits or state they do not exist.

(a) $\displaystyle\lim_{x\to 1} \frac{2x^3 - x^2 + x - 2}{x^4 - 1}$

(b) $\displaystyle\lim_{t\to\infty} \frac{\ln(t)}{t}$

(c) $\displaystyle\lim_{u\to -1} \frac{u^2 + 1}{u + 3}$

(d) $\displaystyle\lim_{x\to 0} \frac{2x\cos(3x)}{\sin(3x)}$

3. (25 points) You're at the beach, and you notice a wave that sort of looks like the function $f(x) = 2x^3 - 3x^2 + 2$. Sketch a graph of $f(x)$ on the interval $[-1, 2]$. Label the x-coordinates of any extreme values and any inflection points[1].

[1] Recall 'inflection points' are the same thing as 'turning points' if you're more familiar with that phrase.

4. (25 points) Farmer Brown wants to build a rectangular pen with four rectangular compartments in it as shown. Fence costs 10 dollars per foot, and he has 1000 dollars to spend on the project. How should he design the pen so it encloses the largest possible area?

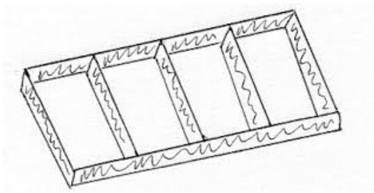


Figure 3: Farmer Brown's pen

5. (25 points) A firework is shot in the air. Its velocity at time t is given by $v(t) = 20 - 6t + 5e^{-t}$, and its initial position is $s(0) = 0$. If the firework bursts at time $t = 2$, then how high up is it when this happens? (i.e., what is $s(2)$?)

6. (25 points) The natural log of 10 is $\ln(10) = 2.30258509\ldots$. Use this and a linear approximation technique to estimate $\ln(10.3)$. Briefly explain if your estimate is larger or smaller than the true answer (provide a crude sketch as needed).

7. (25 points) Alice and Bob are on the boardwalk when they see a carnival ride. The ride is a cart that just moves left and right along a fixed straight-line track. They look at the cart and notice that it is moving to the right. Then they get distracted, and they both look away for a few seconds. When they look back they notice that the cart is in the same spot as it was initially, except now it's moving to the left.

   - Alice says to Bob, "while we weren't looking at that ride, there must have been a moment when the cart wasn't moving."
   - But Bob says, "we can't possibly know that! We weren't looking at the cart at all during that time, so there's no way to tell!"

   Who's right? Use calculus to support your answer. (If you use any results we discussed in class, be sure to reference these.)

8. (30 points) Answer ***one*** of the following questions. If you do more than one, you must circle which question you would like to have graded. For this question, I am just looking for how you would informally explain this to a friend. The idea is for you to demonstrate that you have at least *some* internalization of the concepts. ***Be thorough, but do not write an entire book.***

   (a) What are 'critical points' and what do they have to do with finding maxes and mins? Make a drawing of each type of critical point that shows how that critical point could be the max of a function. Is it the case that *every* critical point is an extreme value? If so, explain why, and if not, draw an example of a critical point that is not an extreme value.

   (b) What's the mean value theorem? Draw a picture that helps illustrate what the mean value theorem states. Draw a function for which the theorem does not apply, and explain why the mean value theorem does not apply in this case.

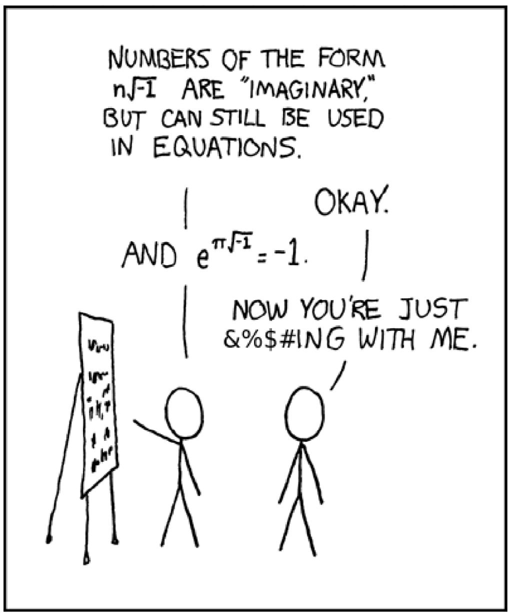
   (c) What is an anti-derivative? When we find anti-derivatives, why is there a '$+C$' that comes up? Why is it necessary? Use pictures and examples to help explain this.

9. **Extra credit 1:** I'm thinking of a super secret function, $f(x)$. I'll let you know that $f(0) = 1$, and I'll also let you know that $f'(x)$ always exists and that $f'(x)$ is never bigger than 1. But that's all I'm saying about my super crazy mystery function. Carefully argue that for every $x > 0$, it must be the case that $f(x) \leq 1 + x$.

Extra credit 2: In class, I mentioned the amazingly beautiful formula $e^{ix} = \cos(x) + i\sin(x)$, which works for any real number x. It turns out that this one formula holds inside it all the trigonometry formulas you've ever seen. For example... Use this formula to prove the "double-angle formulas".

Hint: Start by squaring both sides of the formula to get $e^{i(2x)} = (\cos(x) + i\sin(x))^2$. Simplify the right-hand side just by multiplying it out (remember $i^2 = -1$), and simplify the left-hand side by applying that magical formula again.

Figure 4: Image from xkcd (179, censored)



Formulas Provided for the Exam

This is the sheet of formulas for the exam. This sheet will be (or has been) provided for you exactly as is on the exam. You may not use your own 'formula sheet' for the exam [not even another print-out of this]. During the exam, I will not answer any questions about this formula sheet (since any such questions should have been asked beforehand).

I promise that you will *not* need to use all of these formulas, and that for at least one question, it would benefit you to use at least one. You are expected to memorize the most important formulas for the class (e.g., limit definition of derivative, product rule, Pythagorean theorem, et cetera), and these formulas are therefore not listed here.

Rectangle, circle, triangle: You should know these areas and the circumference of a circle

Trapezoid, bases b_1, b_2, height h: Area $\frac{1}{2}(b_1 + b_2)h$

Rectangular prism (box): You should know (or be able to figure out) volume and surface area

Sphere, radius r: Volume $\frac{4}{3}\pi r^3$; surface area $4\pi r^2$

Cone, radius r, height h: Volume $\frac{1}{3}\pi r^2 h$; surface area (without bottom) $\pi r\sqrt{h^2 + r^2}$

Cylinder, radius r, height h: Volume $h \times$ (*area of circular base*); surface area you should be able to figure out from your knowledge of circles and rectangles