640:300 WORKSHOP 1
SETS, UNIONS AND INTERSECTIONS

Recall that an open interval (a, b) on the real line $\mathbb{R}$ is defined as

$$(a, b) = \{x \in \mathbb{R} \mid a < x < b\}.$$

For a fixed $n \in \mathbb{N} = \{1, 2, 3, 4, \dots\}$, we define the following open interval:

$$S_n = \left(\frac{n-1}{n},\ 1\right).$$

Letting n range over $\mathbb{N}$, we obtain an infinite family of open intervals $\{S_n\}_{n\in\mathbb{N}}$.

(i) Prove that for all $n \in \mathbb{N}$, $S_n \supset S_{n+1}$.

(ii) Prove that $\bigcup_{n\in\mathbb{N}} S_n = (0, 1)$, where

$$\bigcup_{n\in\mathbb{N}} S_n = S_1 \cup S_2 \cup S_3 \cup \dots$$

(iii) What is $|\bigcap_{n\in\mathbb{N}} S_n|$, where

$$\bigcap_{n\in\mathbb{N}} S_n = S_1 \cap S_2 \cap S_3 \cap \dots?$$

Explain your answer.