640:300, Fall 2019
Homework 5
Due Tuesday, Nov. 12, 2019

(1) Prove using the method of smallest counterexamples that for all $n \in \mathbb{N}_{>1}$ we have

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{(n-1)\cdot n}=\frac{(n-1)}{n}.$$

(2) Let $a_1 = 2$, $a_2 = 4$, and $a_{n+2} = 5 \cdot a_{n+1} - 6 \cdot a_n$. Prove using the method of smallest counterexamples that for all $n \in \mathbb{N}$, $a_n = 2^n$.

(3) Prove using the method of smallest counterexamples that for all $n \in \mathbb{N}$

$$3 + 11 + 19 + \cdots + (8n - 5) = 4n^2 - n.$$

(4) Prove using the method of smallest counterexamples that for all $n \in \mathbb{N}$, $5^{2n-1} + 1$ is divisible by 6.

(5) Prove using the method of smallest counterexamples that for all $n \in \mathbb{N}$,

$$\sum_{i=1}^{n} 2^i = 2^{n+1} - 2.$$

(6) Let $\mathbb{Z}_-$ denote the set of all negative integers. Prove using the Well-Ordering Principle that any non-empty subset of $\mathbb{Z}_-$ has a largest element.

(7) Prove by the method of smallest counterexamples that every natural number greater or equal to 11 can be written in the form $2s + 5t$ for some natural numbers s, t.