640:300, Fall 2019
Due Tuesday, Oct. 29, 2019
Homework 4

(1) For $r \in \mathbb{R} \setminus \{1\}$ and $n \in \mathbb{N}$ use induction to show that:

$$1 + r + r^2 + \cdots + r^{n-1} = \frac{1 - r^n}{1 - r}.$$

(2) Prove by induction that for each $n \in \mathbb{N}$, $n^2 < 3^n$.

(3) Prove by induction that for each $n \in \mathbb{N}$, $n^3 + 2n$ is divisible by 3.

(4) Prove by induction that for each $n \in \mathbb{N}$, and for each $x \in \mathbb{R}_{>2}$,

$$x^n + x < x^n \cdot x.$$

(5) Prove by induction that for each $n \in \mathbb{N}$, and for each $x \in \mathbb{R}_{>2}$,

$$nx + 1 < x^n + 2.$$

(Hint: You may need to use Q4 or an equivalent statement in your proof of the inductive step.)

(6) Prove by induction that for each $n \geq 1$, $5^{2n-1} + 1$ is divisible by 6.

(7) Prove the rule of exponents, $(ab)^n = a^n b^n$, for every natural number n, by mathematical induction.

(8) Let $n \geq 1$ be any odd integer. Prove by induction that $n^2 - 1$ is divisible by 4. *(Hint: You cannot prove this by induction directly on n. Rewrite $n^2 - 1$ in terms of a variable on which you can do induction.)*