Math 300, Fall 2019
Homework 3
Due Tuesday Oct. 15, 2019

(1) Find all integer solutions (x, y) to the equation $xy - 3x + 2y = 12$. (*Hint: Rearrange the equation to obtain a factorization.*)

(2) Let n be a positive integer that has 6 and 8 as factors. What other factors must n have?

(3) Let $n \in \mathbb{N}$ and let p be prime. Prove that p cannot divide both n and $n+1$.

(4) Let n be a natural number with prime decomposition $n = p_1^{s_1} p_2^{s_2} \dots p_k^{s_k}$. Prove that if $n = m^2$ for some natural number m then $s_1, s_2, \dots, s_k$ are all even. (*Hint: You can use the Prime Factorization theorem for natural numbers.*)

(5) Let $n \in \mathbb{N}_{>1}$. Prove that if n divides $(n-1)!$ then n has a proper divisor $d > 1$.

(6) Prove that if n is composite, $n = k\ell$, with $1 < k < n$ and $1 < \ell < n$, then k and ℓ both divide $(n-1)!$.

(7) Prove that every prime number $p \in \mathbb{P}_{>3}$ is either of the form of $4n+1$ or of the form of $4n+3$ for some $n \in \mathbb{N}$.

(8) Prove that if $p \in \mathbb{P}_{\geq 5}$ then $p^2 + 2$ is composite.

(9) Let $a, n \geq 2$ be integers. Prove that if $a^n - 1$ is prime, then $a = 2$ and n is prime. (*Hint: use the identity* $x^m - 1 = (x-1)(x^{m-1} + x^{m-2} + \cdots + x + 1)$ *for any integer* $m \geq 2$. *This is another chance to write up the complete solution after the discussion in Workshop 4.*)