

Math 300, Fall 2019
Homework 2
Due Tuesday Oct. 1, 2019

(1) Write the following statements in symbolic language, determine if the statement is true or false (with proof) and negate:

- (a) Every integer of the form $4y + 2$, for some $y \in \mathbb{Z}$, is even.
- (b) There is a positive integer that is prime, divides 6 and divides 9.
- (c) There is a positive integer that is both prime and irrational.

(2) Write the following statements in symbolic language, determine if the statement is true or false (with proof) and negate:

- (a) If a and b are integers with a greater than or equal to 2 then a cannot divide both b and $b + 1$.
- (b) For every positive real number y , there is a unique real number x such that $y = 2^x$.

(3) Prove that $P \implies Q$ and $\neg Q \implies \neg P$ are logically equivalent but that $P \implies Q$ and $Q \implies P$ are not logically equivalent.

(4) Prove that $\neg(P \wedge Q)$ and $(P \wedge \neg Q) \vee \neg P$ are logically equivalent.

(5) Prove that $P \iff Q$ is logically equivalent to $(P \implies Q) \wedge (Q \implies P)$.

(6) Prove by contradiction that there is no integer k such that $4k + 3 = m^2$ for some $m \in \mathbb{Z}$.

Hint: Use the fact that $4k + 3$ is odd.

(7) Prove by contradiction that if $a, b \in \mathbb{Z}$ then $a^2 - 4b \neq 2$.

(8) Prove by contradiction that for $a, b \in \mathbb{Z}$, if ab^2 is even, then a is even or b is even.