640:300 Intro Math Reasoning
Problem Set 1: Sets and number systems
Due Tuesday Sep. 17, 2019

Question 1. Let A, B be subsets of a set X. Prove that $A \subseteq B$ if and only if $B^c \subseteq A^c$.

Question 2. Let

$$S = \left\{ \frac{a}{2^n} \mid a \in \mathbb{Z}_{\geq 0}, n \in \mathbb{Z}_{>0} \right\} \subset \mathbb{Q}$$

$$T = \left\{ \frac{b}{3^m} \mid b \in \mathbb{Z}_{\geq 0}, m \in \mathbb{Z}_{>0} \right\} \subset \mathbb{Q}.$$

Find $S \cap T$.

Question 3. Does every integer have a multiplicative inverse? That is, for each integer x, can you find an integer y such that $xy = 1$? Explain your answer.

Question 4. Prove that the sum of two even integers is even.

Question 5. Prove that the product of any integer with an even integer is even.

Question 6. Let $a, b, c \in \mathbb{Z}$ with $a, b \neq 0$. Prove that if a divides b, and b divides c, then a divides c.

Question 7. Let $S = \{x \in \mathbb{Z} \mid x = 2y - 3 \text{ for some } y \in \mathbb{Z}\}$. Prove that $S = \mathbb{O}$, where $\mathbb{O}$ denotes the set of odd integers.

Question 8. Prove that an integer n is divisible by 6 if and only if it is divisible by both 2 and 3.

Question 9. Let a, b be integers with $b \neq 0$. Suppose that the quadratic equation $x^2 + ax + b = 0$ has solutions z and w. If $z \in \mathbb{Z}$ and $w \in \mathbb{Z}$, Prove that z divides b and w divides b.

Question 10. Do there exist pairs (x, y) of integers that satisfy $x + 5y = 10$? Justify your answer.