640:300 WORKSHOP 9
EQUIVALENCE RELATIONS AND EQUIVALENCE CLASSES

(1) Define a relation '$\backsim$' on $\mathbb{R}$ by

$$x \backsim y \quad \Longleftrightarrow \quad x = y \text{ or } xy = 1$$

for all $x, y \in \mathbb{R}$.

    (i) Show that $\backsim$ is an equivalence relation.

    (ii) Find the equivalence classes of (a) 5; (b) $-2/5$; (c) 0.

(2) Let X be the set of all cities in the US. Define a relation $\sim$ on X by

$$x \sim y \quad \Longleftrightarrow \quad x \text{ and } y \text{ lie in the same state}$$

for all $x, y \in X$. (*We assume that each city in the US lies in a unique state.*)

    (i) Show that $\sim$ is an equivalence relation

    (ii) Find the partition of X under $\sim$.

(3) Find the intersection $X \cap Y$, where X is the equivalence class of 1 under congruence modulo 2 and Y is the equivalence class of 2 under congruence modulo 3.