

640:300 WORKSHOP 5
EQUATIONS WITH INTEGER SOLUTIONS

The following problems concern certain *Diophantine equations*. These are equations for which we seek only integer solutions.

(i) Prove that there are no integers x and y such that: $x > 0$ and $y > 0$

$$x^2 - y^2 = 2.$$

(ii) Let $x, y, z \in \mathbb{Z}$. Prove that if $xyz = 1$, then the only solutions for (x, y, z) are:

$$(1, 1, 1), (-1, -1, 1), (1, -1, -1), (-1, 1, -1).$$

Hint. This is a Diophantine equation involving 3 variables. Modify it so that you would deal with two (possibly new) variables.