640:300 WORKSHOP 2
ODD AND EVEN INTEGERS

We follow Euclid's definitions of even numbers (The Elements: Book VII: Def. 6) and odd numbers (The Elements: Book VII: Def. 7):

An *even number* is that which is divisible into two equal parts.

Let $\mathbb{E}$ denote the set of even integers. We may write

$$\mathbb{E} = \{x \in \mathbb{Z} \mid (\exists y \in \mathbb{Z})(x = 2y)\}.$$

Then

$$\mathbb{E} = \{\cdots - 6, -4, -2, 0, 2, 4, 6 \dots\}.$$

An *odd number* is that which is not divisible into two equal parts. Let $\mathbb{O}$ denote the set of odd integers. Then

$$\mathbb{O} = \mathbb{Z} - \mathbb{E} = \{x \in \mathbb{Z} \mid x \notin \mathbb{E}\}$$

and

$$\mathbb{O} = \{\cdots - 5, -3, -1, 1, 3, 5 \dots\}.$$

Prove the following theorem.

Theorem Let $\mathbb{O}' = \{z \in \mathbb{Z} \mid (\exists y \in \mathbb{Z})(z = 2y + 1)\}$. Then $\mathbb{O}' \subseteq \mathbb{O}$.

Recall that to prove $\mathbb{O}' \subseteq \mathbb{O}$ we must prove that $x \in \mathbb{O}' \implies x \in \mathbb{O}$.

Hint: Use proof by contradiction.