Quantum Theory Without Observers

Sheldon Goldstein
Department of Mathematics, Rutgers University
New Brunswick, NJ 08903, USA

October 1, 1997

> The concept of 'measurement' becomes so fuzzy on reflection that it is quite surprising to have it appearing in physical theory *at the most fundamental level.* ... [D]oes not any *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts? (Bell 1981 [1, page 117])

1 Introduction

Despite its extraordinary predictive successes, quantum mechanics has, since its inception some seventy years ago, been plagued by conceptual difficulties. The basic problem, plainly put, is this: It is not at all clear what quantum mechanics is about. What, in fact, does quantum mechanics describe?

It might seem, since it is widely agreed that the state of any quantum mechanical system is completely specified by its wave function, that quantum mechanics is fundamentally about the behavior of wave functions. Quite naturally, no physicist wanted this to be true more than did Erwin Schrödinger, the father of the wave function. Nonetheless, Schrödinger ultimately found this impossible to believe. His difficulty was not so much with the novelty of the wave function [2, page 156 of [3]]: "That it is an abstract, unintuitive mathematical construct is a scruple that almost always surfaces against new aids to thought and that carries no great message." Rather, it was that the "blurring" suggested by the spread out character of the wave function "affects macroscopically tangible and visible things, for which the term 'blurring' seems simply wrong."

For example, Schrödinger noted that it may happen in radioactive decay that "the emerging particle is described ... as a spherical wave ... that impinges continuously on a surrounding luminescent screen over its full expanse. The screen however does not show a more or less constant uniform surface glow, but rather lights up at *one* instant at *one* spot...." And he observed that one can easily arrange, for example by including a cat in the system, "quite ridiculous cases" with "the ψ-function of the entire system having in it the living and the dead cat (pardon the expression) mixed or smeared out in equal parts."

It is thus because of the "measurement problem," of macroscopic superpositions, that Schrödinger found it difficult to regard the wave function as "representing reality." But then what does? With evident disapproval, Schrödinger describes how "the reigning doctrine rescues itself or us by having recourse to epistemology. We are told that no distinction is to be made between the state of a natural object and what I know about it, or perhaps better, what I can know about it if I go to some trouble. Actually—so they say—there is intrinsically only awareness, observation, measurement."

Schrödinger's portrayal of the views of his contemporaries was quite accurate. Niels Bohr [4, page 210], the founder of the "Copenhagen interpretation," insisted upon the "*impossibility of any sharp separation between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear*" and claimed [4, page 235] that "in quantum mechanics, we are not dealing with an arbitrary renunciation of a more detailed analysis of atomic phenomena, but with a recognition that such an analysis is *in principle* excluded." Werner Heisenberg [6, page 129] claimed that "the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them ... is impossible ... " and that [7, page 15] "We can no longer speak of the behavior of the particle independently of the process of observation. As a final consequence, the natural laws formulated mathematically in quantum theory no longer deal with the elementary particles themselves but with our knowledge of them. Nor is it any longer possible to ask whether or not these particles exist in space and time objectively."

Many physicists pay lip service to the Copenhagen interpretation, and in particular to the notion that quantum mechanics is about observation or results of measurement. But hardly anybody truly believes this anymore—and it is hard for me to believe anyone really ever did. It seems clear that quantum mechanics is fundamentally about atoms and electrons, quarks and strings, and not primarily about those particular macroscopic regularities associated with what we call measurements of the properties of these things. But this, of course, does not really provide an answer to the question with which I began. After all, if these entities are not to be somehow identified with the wave function itself—and if talk of them is not merely shorthand for elaborate statements about measurements—then where are they to be found in the quantum description?

There is, perhaps, a very simple reason why there has been so much difficulty discerning in the quantum description the objects we believe quantum mechanics should be describing. Perhaps the quantum mechanical description is not the whole story, a possibility most prominently associated with Albert Einstein.

On the basis of more or less the same considerations as those of Schrödinger quoted above, Einstein concluded that the wave function does not provide an exhaustive description of individual systems, while noting [8, page 672] that "there exists ... a simple psychological reason for the fact that this most nearly obvious interpretation is being shunned. For if the statistical quantum theory does not pretend to describe the individual system ... completely, it appears unavoidable to look elsewhere for a complete description of the individual system." In relation to this more complete theory, "the statistical quantum theory would ... take an approximately analogous position to the statistical mechanics within the framework of classical mechanics." Earlier, Einstein, Boris Podolsky and Nathan Rosen concluded their famous EPR paper [9] as follows: "While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible."

Regarded as a response to the measurement problem, the position of Bohr and Heisenberg seems excessive in comparison with that of Einstein. After all, the latter denied merely that the wave function is a complete description of an observer-independent physical reality, while the former seemed to deny that there is any such reality, at least insofar as atomic phenomena are concerned! And as regards the plausibility of their conclusions, Einstein's insistence on the possibility of a more complete description seems rather modest when contrasted with the categorical assertions of "impossibility" and "in principle" exclusion of Bohr. Nonetheless, it is generally believed in the physics community that Bohr vanquished Einstein in their great, decades-long, debate. At the same time, it is also widely believed that their debate was merely philosophical and hence not susceptible to any clear cut resolution.

However, the Bohr-Einstein debate has already been resolved, and in favor of Einstein: What Einstein desired and Bohr deemed impossible—an observer-free formulation of quantum mechanics, in which the process of measurement can be analyzed in terms of more fundamental concepts—does, in fact, exist. Moreover, there are many such formulations, the most promising of which belong to three basic categories or approaches: decoherent histories, spontaneous localization, and pilot-wave theories. The simplest pilot-wave theory, Bohmian mechanics, has in fact existed almost since the inception of quantum theory itself. These approaches can be regarded, each in its own way, as minimal responses to the problem of formulating a quantum theory without observers. Each of these, I will argue, can also be regarded as realizations of

Einstein's insight that the wave function does not provide us with a complete description of physical reality, and of his belief that a more complete theory is possible.

2 Decoherent Histories

The decoherent histories (DH) approach was initiated in 1984 by Robert Griffiths [10] (who spoke, however, of consistent histories) and independently proposed by Roland Omnès [11] a little later; it was subsequently rediscovered by Murray Gell-Mann and James Hartle [12], who made crucial contributions to its development. (DH should not be confused with the environment-induced superselection approach of Wojciech Zurek [13], in which decoherence also plays a crucial role, but one that differs significantly from its role in DH. In this approach the environment is fundamental, acting in effect as an observer, so that it is difficult to regard this proposal as genuinely providing a quantum theory without observers.)

DH may be regarded as a minimalist approach to the conversion of the quantum measurement formalism to a theory governing sequences of objective events, including, but not limited to, those that we regard as directly associated with measurements. Where the Copenhagen interpretation talks about *finding* (and thereby typically disturbing) such and such observables with such and such values at such and such times, the decoherent histories approach speaks of such and such observables *having* such and such values at such and such times. To each such *history* h, DH assigns the same probability $P(h)$ of *happening* that the quantum measurement formalism—the wave function reduction postulate for ideal measurements together with the Schrödinger evolution—would assign to the probability of *observing* that history in a sequence of ideal (coarse-grained) measurements of the respective observables at the respective times: If the (initial) wave function of the system is ψ,

$$P(h) = \langle E(h)\psi | E(h)\psi \rangle \tag{1}$$

where $E(h) = E_n \cdots E_2 E_1$ with $E_1, E_2, \ldots, E_n$ the Heisenberg projection operators corresponding to the time-ordered sequence of events defining the history h. For example, for a spin 1/2 particle initially (at $t = 0$) in the state $\psi = | \uparrow \rangle_z$ with $\sigma_z = 1$, we might consider the history h for which $\sigma_x = 1$ at $t = 1$ and $\sigma_y = -1$ at $t = 2$. For Hamiltonian $H = 0$, (1) then yields $P(h) = 1/4$.

DH can be regarded as describing a stochastic process, a process with intrinsic randomness. Think, for example, of a random walk, with histories corresponding to a sequence of jumps and probabilities of histories generated by the probabilities for the individual jumps. The histories with which DH is concerned are histories of observables, for example of positions of particles.

While Schrödinger's spherical wave impinges continuously on a screen over its full expanse, the screen lights up at one instant at one spot because it is precisely with such events that DH is concerned and to such events that DH assigns nonvanishing probability.

To understand DH one must appreciate that the histories with which it is concerned are not histories of wave functions. For DH the wave function is by no means the complete description of a quantum system; it is not even the most important part of that description. DH is primarily concerned with histories of observables, not of wave functions, which play only a secondary role, as a theoretical ingredient in the formulation of laws governing the evolution of quantum observables via the probabilities assigned to histories. Thus DH avoids the measurement problem in exactly the manner suggested by Einstein.

It should come as no surprise that the consistent development of the DH idea, of assigning probabilities to objective histories, is not so easy to achieve. After all, Bohr and Heisenberg were no fools; they surely would not have insisted that all is observation were such a radical conclusion easily avoidable. It is only as a first approximation that DH can be regarded as merely describing a stochastic process. There are, in fact, some very significant differences. Perhaps the most crucial of these concerns the role of coarse graining. Because of quantum interference effects, coarse graining plays an essential role for DH, not just for the description of events of interest to us, but in the very formulation of the theory itself. A fine-grained history—given for a system of particles by, for example, the precise specification of the positions of all particles at all times in some time-interval—will normally not be assigned any probability. In fact, most coarse-grained histories won't either.

For example, for the two-slit experiment DH assigns no probability to the history in which the particle passes (unobserved) through, say, the upper slit and lands in a small neighborhood of a specific point on the scintillation-screen. Nor, indeed, does it assign any probability to the spin history that differs from the one described after formula (1) only by the replacement of $\sigma_y = -1$ by $\sigma_z = -1$ at $t = 2$. This is because (1) yields the value 1/4 also for this history, which is inconsistent with the value 0 for the corresponding coarse-grained history with $t = 1$ ignored! (These values involve no inconsistency for the usual quantum theory, in which they concern the results of measurements, since the *measurement* of σ_x at $t = 1$ would be expected to disturb σ_z.)

DH assigns probabilities, via (1), only to histories belonging to special families $\mathcal{H}$, closed under coarse graining, which satisfy a certain *decoherence condition* (DC)

$$\mathrm{Re}\langle E(h)\psi|E(h')\psi\rangle = 0 \quad \text{for all } h, h' \in \mathcal{H} \text{ with } h \neq h' \tag{2}$$

guaranteeing that $P(h)$ is additive on $\mathcal{H}$ and hence provides a consistent assignment of probabilities to elements of $\mathcal{H}$. (The decoherence condition in fact has several versions, the differences between which I shall here ignore. There is also a perhaps simpler version of (1), with a linear dependence on $E(h)$, that involves a much more robust decoherence condition than DC [14].) Whether or not a family $\mathcal{H}$ satisfies the DC depends not only on a sequence of times and coarse-grained observables at those times, but also upon the (initial) wave function ψ (or density matrix ρ) as well as the Hamiltonian H of the relevant system, so it is convenient to regard also these as part of the specification of $\mathcal{H}$. DH thus assigns probabilities $P(h)$ to those histories h that belong to at least one decoherent family $\mathcal{H}$ (as I will call those families satisfying the DC).

It turns out, naturally enough, that a family of histories describing the results of a sequence of measurements will normally be decoherent, regardless of whether or not we actually observe the measurement devices involved. Moreover, interaction with a measurement device is incidental; satisfaction of the DC may be induced by any suitable interaction—or by none at all.

In fact, families defined by conditions on (commuting) observables at a *single* time are always decoherent. After all, it is for precisely such families that textbook quantum mechanics supplies perfectly straightforward probability formulas—via spectral measures. It is important to bear in mind, however, that even for such standard families, the textbook probability formulas have an entirely different meaning for DH than for orthodox quantum theory, describing the probability distribution of the *actual* value of the relevant observable at the time under consideration, and not merely the distribution of the value that would be found were the appropriate measurement performed. This difference is the source of a very serious difficulty for DH.

The difficulty arises already for the standard families, involving observables at a single time. The problem is that the way that the probabilities $P(h)$ are intended in the DH approach, as probabilities of what objectively happens and not merely of what would be observed upon measurement, is precisely what is precluded by the no-hidden-variables theorems of, e.g., Gleason [15, 25, 33], Kochen and Specker [16, 33], or Bell [17, 25, 33]! It is a consequence of these theorems that the totality of joint quantum mechanical probabilities for the various sets of commuting observables is genuinely inconsistent: the ascription of these probabilities to actual joint values, as relative frequencies over an ensemble of systems—a single ensemble, defined by the wave function under consideration, for the totality—involves a contradiction, albeit a hidden one. For example, the correlations between spin components for a pair of spin 1/2 particles in the singlet state, if consistent, would have to satisfy Bell's inequality. They don't.

A simple and dramatic example of the sort of inconsistency I have in mind was recently found by Lucien Hardy [18]. For almost all spin states of a pair of spin 1/2 particles[1] there are spin components A, B, C, and D such that the quantum probabilities for appropriate pairs would imply that in a large ensemble of such systems (1) it sometimes happens that $A = 1$ and also $B = 1$; (2) whenever $A = 1$, also $C = 1$; (3) whenever $B = 1$, also $D = 1$; and (4) it never happens that $C = 1 = D$. The quantum probabilities are thus inconsistent: there clearly is no such ensemble. (The probability that $A = 1 = B$ is about 9% with optimal choices of state and observables.) The inconsistency implied by violation of Bell's inequality is of a similar nature.

Thus, as so far formulated, DH is not well defined. For a given system, with specified Hamiltonian and fixed initial wave function, the collection of numbers $P(h)$, with h belonging to at least one decoherent family, cannot consistently be regarded as intended—as the probability for the occurrence of h. Too many histories have been assigned probabilities: to be well defined, DH must restrict, by some further condition or other, the class of decoherent families whose elements are to be assigned probabilities. It is not absolutely essential that there be only one such family. But if there be more than one, it is essential that the probabilities defined on them be mutually consistent.

Without directly addressing this problem of mutual inconsistency, Gell-Mann and Hartle [12, 19] have emphasized that for various reasons the DC alone allows far too many families. They have therefore introduced additional conditions on families, such as "fullness" and "maximality," and have proposed distinguishing families according to certain tentative measures of "classicity." With such ingredients they hope to define an optimization procedure—and hence what might be called an optimality condition—that will yield a possibly unique "quasiclassical domain of familiar experience," a family that should be thought of as describing familiar (coarse-grained) macroscopic variables, for example hydrodynamical variables. When the probability formula $P(h)$ is applied to this family, it is hoped that the usual macroscopic laws, including those of phenomenological hydrodynamics, will emerge, together with quantum corrections permitting occasional random fluctuations on top of the deterministic macrolaws (and classical fluctuations).

GMH do not seem to regard their additional conditions as fundamental, but rather merely as ingredients crucial to their analysis of a theory they believe already defined by the DC alone. While I've argued that there is no such theory, a physical theory could well be defined by the decoherence condition together with suitable additional conditions (DC+) like those proposed by GMH, also regarded as fundamental. In this way, DH becomes a serious program for the construction of a quantum theory without observers.

[1]The exceptions are the product states and, perhaps surprisingly, maximally entangled states like the singlet state.

It is true that much work remains to be done in the detailed construction of a theory along these lines, even insofar as nonrelativistic quantum mechanics is concerned. It is also true that many questions remain concerning exactly what is going on in a universe governed by DH, particularly with regard to the irreducible coarse graining. Nonetheless it seems likely that the program of DH can be brought successfully to completion. It is, however, not at all clear that the theory thus achieved will possess the simplicity and clarity expected of a fundamental physical theory. The approach to which I shall now turn has already led to the construction of several precise and reasonably simple versions of quantum theory without observers.

3 Spontaneous Localization

The spontaneous localization (SL) approach, initiated by Philip Pearle around 1970, may be regarded as concerned with a minimal modification of the Schrödinger evolution into one in which wave functions of macroscopic systems behave in a sensible way. This goal proved elusive, but in 1985 a breakthrough [20] occurred: GianCarlo Ghirardi, Alberto Rimini, and Tulio Weber (GRW), by appreciating the privileged role somehow played by positions and thus focusing on the possibility of *spatial* localization, showed how to combine the Schrödinger evolution with spontaneous random collapses—given by "Gaussian hits" centered at random positions $\mathbf{x}$ occurring at random times t—to obtain an evolution for wave functions that reproduces the Schrödinger evolution on the atomic level while avoiding the embarrassment of macroscopic superpositions.

Thus [1, page 204] "any embarrassing macroscopic ambiguity in the usual theory is only momentary in the GRW theory. The cat is not both dead and alive for more than a split second." Similarly, measurement pointers quickly point. Moreover, it is a more or less immediate consequence of the GRW dynamics that when a macroscopic superposition $\psi = \sum_\alpha \psi_\alpha$ collapses under the GRW evolution to one of its terms, the probability that ψ_α is the term that survives is $\|\psi_\alpha\|^2$, precisely as demanded by the collapse postulate of standard quantum theory.

It is tempting to say that with the SL approach, quantum mechanics is indeed fundamentally about the behavior of wave functions. I believe, however, that this is not quite right. The problem is that the purpose of any physical theory is to account for a pattern of events occurring in (ordinary 3-dimensional) space and time. But the behavior of a wave function of a many (N) particle universe, a field on an abstract ($3N$-dimensional) configuration space, has in and of itself no implications whatsoever regarding occurrences in physical space, however sensible its behavior may otherwise be. As Bell [1, page 204] has noted "it makes no sense to ask for the amplitude or phase or whatever of the wave function at a point in ordinary space. It has neither amplitude nor phase nor anything else until a multitude of points in ordinary three-space are specified."

Therefore Ghirardi [21, page 8] rightly emphasizes the importance of specifying what he calls "the physical reality of *what exists out there*." For this he chooses the mass density function, which for the simple GRW theory described here can be identified with the mass weighted sum $\sum_i m_i \rho_i(\mathbf{x})$, over all particles, of the one-particle densities ρ_i arising from integrating $|\psi|^2$ over the coordinates of all but one of the particles.[2]

Bell [1, page 205] has proposed a strikingly different possibility,[3] that the space-time points $(\mathbf{x}, t)$ at which the hits are centered (which are determined by the wave function trajectory) should themselves serve as the "local beables of the theory. These are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of physical theories, as distinct from things which may be real but not localized, and as distinct from the 'observables' of other formulations of quantum mechanics, for which we have no use here.) A piece of matter then is a galaxy of such events."

One can imagine, of course, many other choices, some better than others. The point I wish to emphasize here, however, is that if we are to have a well-defined physical theory at all, some such choice must be made. Indeed, any quantum theory without observers, and arguably any physical theory with any pretense to precision, requires as part of its formulation a specification of the "local beables," of "what exists out there," of what the theory is fundamentally about—which I would prefer to call the *primitive ontology* of the theory. (It might be argued that the unease sometimes expressed about DH arises from the obscurity of its primitive ontology—or from its failure to commit in this regard.) Moreover, we must also specify, for a quantum theory, the relationship between the wave function and this primitive ontology, which for SL will be provided by a mapping or code connecting the evolution of the wave function to a story in space and time.

Different such specifications define different theories. They might also have different observable consequences. Moreover, the symmetries of the theory may depend critically on this specification. For example with Bell's rather surprising choice the GRW theory obeys a certain "relative time translation invariance" and becomes [1, page 209] "as Lorentz invariant as it could be in the nonrelativistic version." Thus a careful analysis of the symmetries of a theory demands a careful specification of its primitive ontology.

[2] Because of subtle considerations related to the notion of "accessibility," Ghirardi's specific choice is actually the mass density averaged over a "localization volume."

[3] Bell's proposal is not applicable to models involving continuous dynamical reduction [21].

As a matter of fact, one would have to make a rather perverse choice to arrive at any empirical disagreement with the predictions arising from the choices of Ghirardi or Bell. It is clear, however, because of its abrogation of the Schrödinger evolution, that SL (in whatever version and with whatever choice of primitive ontology) must disagree somewhat with the predictions of orthodox quantum theory. In fact, by the uncertainty principle, the wave function localizations will increase the momentum space spread in the wave function and hence energy will tend to increase at a very small rate—so small in fact that this effect may be rather difficult to observe.

The last version of quantum theory without observers that I shall describe agrees completely with orthodox quantum theory in its predictions. Precise and simple, it involves an almost obvious incorporation of Schrödinger's equation into an entirely deterministic reformulation of quantum theory.

4 Bohmian Mechanics

In the pilot-wave (or Bohmian) approach, quantum theory is fundamentally about the behavior of particles, described by their positions—or fields, described by field configurations, or strings, described by string configurations—and only secondarily about wave functions. In this approach the wave function, obeying Schrödinger's equation, does not provide a complete description or representation of a quantum system. Instead, the wave function choreographs or governs the motion of the more fundamental variables.

Bohmian mechanics (or the de Broglie-Bohm theory) is the minimal completion of Schrödinger's equation, for a nonrelativistic system of particles, to a theory describing a genuine motion of particles. For Bohmian mechanics the state of the system is described by its wave function $\psi = \psi(\mathbf{q}_1, \ldots, \mathbf{q}_N)$ together with the configuration Q defined by the positions $\mathbf{Q}_1, \ldots, \mathbf{Q}_N$ of its particles. The theory is then defined by two evolution equations: Schrödinger's equation for $\psi(t)$ and a first-order evolution equation[4]

$$\frac{d\mathbf{Q}_k}{dt} = \mathbf{v}_k^{\psi}(\mathbf{Q}_1, \ldots, \mathbf{Q}_N) \equiv \frac{\hbar}{m_k}\mathrm{Im}\frac{\psi^* \nabla_{\mathbf{q}_k}\psi}{\psi^*\psi}(\mathbf{Q}_1, \ldots, \mathbf{Q}_N) \tag{3}$$

for $Q(t)$, the simplest first-order evolution equation for the positions of the particles that is compatible with the Galilean (and time-reversal) covariance of the Schrödinger evolution [24, pages 852–854]. Here m_k is the mass of the k-th particle. (If ψ is spinor-valued, the products

[4]Since the denominator on the right hand side of (3) vanishes at the nodes of ψ, global existence and uniqueness for the Bohmian dynamics is a nontrivial matter. It is proven in [22].

in numerator and denominator should be understood as scalar products. If external magnetic fields are present, the gradient should be understood as the covariant derivative, involving the vector potential.) This deterministic theory of particles in motion completely accounts for all the phenomena of nonrelativistic quantum mechanics, from interference effects to spectral lines [23, pages 175–178] to spin [25, page 10 of [1]], and it does so in a completely ordinary manner.

Note that given an initial wave function ψ_0, the full Bohmian trajectory $Q(t)$ is determined by the initial configuration Q_0. Thus, given any probability distribution for the initial configuration, Bohmian mechanics defines a probability distribution for the full trajectory. Moreover, since the right hand side of (3) is J/ρ, where J is the quantum probability current and ρ is the quantum probability density, it follows from the quantum continuity equation $\partial\rho/\partial t + \operatorname{div} J = 0$ that if the distribution of the configuration Q is given by $|\psi|^2$ at some time (say the initial time) this will be true at all times. Thus Bohmian mechanics provides us with probabilities for completely fine-grained configurational histories that are consistent with the quantum mechanical probabilities for configurations, including the positions of instrument pointers, at single times.

The pilot-wave approach to quantum theory was initiated, even before the discovery of quantum mechanics itself, by Einstein, who hoped that interference phenomena involving particle-like photons could be explained if the motion of the photons were somehow guided by the electromagnetic field—which would thus play the role of what he called a *Führungsfeld* or guiding field [26, page 262]. While the notion of the electromagnetic field as guiding field turned out to be rather problematical, the possibility that for a system of electrons the wave function might play this role, of guiding field or pilot wave, was explored by Max Born in his early paper founding quantum scattering theory [27]—a suggestion to which Heisenberg was profoundly unsympathetic.

By 1927, an equation of particle motion equivalent to (3) for a scalar wave function had been written down by Louis de Broglie [28, page 119], who explained at the 1927 Solvay Congress how this motion could account for quantum interference phenomena. However, de Broglie badly failed to respond adequately to an objection of Wolfgang Pauli [30, pages 280–282] concerning inelastic scattering, no doubt making a rather poor impression on the illustrious audience gathered for the occasion (see Figure 7).

Born and De Broglie very quickly abandoned the pilot-wave approach and became enthusiastic supporters of the rapidly developing consensus in favor of the Copenhagen interpretation. Bohmian mechanics was rediscovered in 1952 by David Bohm, the first person genuinely to understand its significance and implications. (Unfortunately, Bohm's formulation involved unnecessary complications and could not deal efficiently with spin. In particular, Bohm's invocation of the "quantum potential" made his theory seem artificial and obscured its essential structure. For more on this, see [31].) Its principal advocate during the past three decades was Bell [1]. Impelled by the evident nonlocality of Bohmian mechanics, Bell [17] established, using the "no-hidden-variables theorem" based on his famous inequality, that this feature was unavoidable by any serious theory accounting for the quantum predictions.

The possibility of a deterministic reformulation of quantum theory such as Bohmian mechanics has been regarded by almost all luminaries of quantum physics as having been conclusively refuted. For several decades this refutation was believed to have been provided by the 1932 no-hidden-variables proof of John von Neumann [32], despite the fact that, according to Bell [33], von Neumann's assumptions are so unreasonable that "the proof of von Neumann is not merely false but *foolish!*" While some physicists continue to rely on von Neumann's proof, it is interesting to note that in recent years it is more common to find physicists citing Bell's no-hidden-variables theorem as the basis of this refutation—thus failing to appreciate that what Bell demonstrated with his theorem was not the impossibility of Bohmian mechanics but rather that its most radical implication, namely nonlocality, was intrinsic to quantum theory itself.

According to Richard Feynman, the two-slit experiment for electrons is [34, page 37–2] "a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the *only* mystery." This experiment [35, page 130] "has been designed to contain all of the mystery of quantum mechanics, to put you up against the paradoxes and mysteries and peculiarities of nature one hundred per cent." As to the question [35, page 145], "How does it really work? What machinery is actually producing this thing? Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it."

But Bohmian mechanics is just such a deeper explanation (as is SL, of which, however, Feynman could not have been aware). It resolves the dilemma of the appearance, in one and the same phenomenon, of both particle and wave properties in a rather straightforward manner: Bohmian mechanics is a theory of motion describing a particle (or particles) guided by a wave. In Figure 1 we have a family of Bohmian trajectories for the two-slit experiment. While each trajectory passes through but one of the slits, the wave passes through both; the interference profile that therefore develops in the wave generates a similar pattern in the trajectories guided by this wave.

Compare Feynman's presentation with that of Bell [1, page 191]:

> Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in screen, could be influenced by waves propagating through both holes. And so influenced that

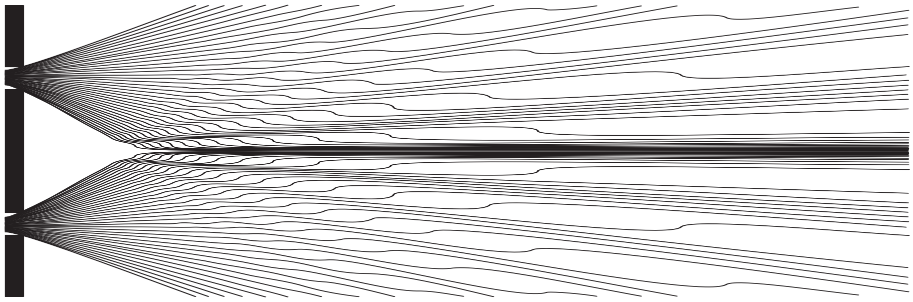

Figure 1: An ensemble of trajectories for the two-slit experiment, uniform in the slits. (Drawn by G. Bauer from [36].)

> the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored. (Bell 1986)

Nonetheless, it would appear that inasmuch as orthodox quantum theory supplies us with probabilities not merely for positions but for a huge class of quantum observables, it is a much richer theory than Bohmian mechanics, which seems exclusively concerned with positions. Appearances are, however, misleading. In this regard, as with so much else in the foundations of quantum mechanics, the crucial observation has been made by Bell [1, page 166]:

> ...in physics the only observations we must consider are position observations, if only the positions of instrument pointers. It is a great merit of the de Broglie-Bohm picture to force us to consider this fact. If you make axioms, rather than definitions and theorems, about the 'measurement' of anything else, then you commit redundancy and risk inconsistency. (Bell 1982)

Bell's point here is well taken: the usual measurement postulates of quantum theory, including collapse of the wave function and probabilities for measurement results given by the absolute square of probability amplitudes, emerge [23] as soon as we take seriously the equations of Bohmian mechanics and what they describe—provided that the initial configuration of a system is random, with probability distribution given by $\rho = |\psi|^2$. Moreover, Detlef Dürr, Nino Zanghì, and I have shown [24] how probabilities for positions given by $|\psi|^2$ emerge naturally from an analysis of "equilibrium" for the deterministic dynamical system defined by Bohmian mechanics, in much the same way that the Maxwellian velocity distribution emerges from an analysis of classical thermodynamic equilibrium. Thus with Bohmian mechanics the statistical description in quantum theory indeed takes, as Einstein anticipated, "an approximately analogous position to the statistical mechanics within the framework of classical mechanics."

5 Reality and the Role of the Wave Function

Bohmian mechanics is, it seems to me, by far the simplest and clearest version of quantum theory. Nonetheless, with its additional variables and equations beyond those of standard quantum mechanics, Bohmian mechanics has seemed to most physicists to involve too radical a departure from quantum modes of thought. The approaches of spontaneous localization and decoherent histories have achieved much wider acceptance among physicists, the former because it ostensibly involves only wave functions, effectively collapsing upon measurement in the usual text-book manner, and the latter because it apparently is defined solely in terms of standard quantum mechanical machinery—i.e., the quantum measurement formulas of the orthodox theory, involving wave functions and sequences of Heisenberg projection operators.

However, SL clearly involves equations beyond those of orthodox quantum theory, and, as I've argued, DH must also be regarded in this way. I have also argued that neither for DH nor even for SL can the wave function be regarded as providing the complete description of a physical system. Thus, while there are significant differences in detail, the three approaches discussed here have much more in common than is usually acknowledged. Each involves additional equations and additional variables; the latter are the fundamental variables, describing the primitive ontology—what the theory is fundamentally about. The behavior of the fundamental variables is governed by laws expressed in terms of the wave function, which thus simply plays a dynamical role.

As to detail, Bohmian mechanics shows that if we don't insist upon patterning these laws upon familiar formulas such as those of the quantum measurement formalism, surprising simplicity can be achieved. GRW, particularly a la Bell, shows that these laws may be of a most unusual variety, with unexpected implications for the symmetry of the theory [1, page 209]. And DH introduces a fundamental irreducible coarse-graining and, if it should turn out that more than one family satisfies DC+, suggests that a fundamental stochastic theory need not assign probabilities to everything that can happen—for example, to histories of the form "h and $\tilde{h}$" where h and $\tilde{h}$ belong to different DC+ families, while the history "h and $\tilde{h}$" belongs to no such family.

None of the theories sketched here is Lorentz invariant. There is a good reason for this: the intrinsic nonlocality of quantum theory presents formidable difficulties for the development of a Lorentz invariant formulation that avoids the vagueness of the orthodox version. I believe, however, that such a theory is possible, and that the three approaches I've discussed here have much to teach us about how we might go about finding one.

Acknowledgments

I am grateful to Karin Berndl, Jean Bricmont, Martin Daumer, Detlef Dürr, GianCarlo Ghirardi, Rebecca Goldstein, Michael Kiessling, Joel Lebowitz, Eugene Speer, Herbert Spohn, and Nino Zanghì for their comments and suggestions—as well as for their patience. This work was supported in part by NSF Grant No. DMS-9504556.

References

[1] J. S. Bell, *Speakable and unspeakable in quantum mechanics,* Cambridge University Press, Cambridge (1987).

[2] E. Schrödinger, Die gegenwärtige Situation in der Quantenmechanik, Naturwissenschaften **23**, 807 (1935); English translation by J. D. Trimmer, The Present Situation in Quantum Mechanics: A Translation of Schrödinger's "Cat Paradox" Paper, Proceedings of the American Philosophical Society **124**, 323 (1980), reprinted in [3].

[3] J. A. Wheeler and W. H. Zurek, eds., *Quantum Theory and Measurement,* Princeton University Press, Princeton (1983).

[4] N. Bohr, Discussion with Einstein on Epistemological Problems in Atomic Physics, in [5].

[5] P. A. Schilpp, ed., *Albert Einstein, Philosopher-Scientist,* Library of Living Philosophers, Evanston, IL, (1949).

[6] W. Heisenberg, *Physics and Philosophy,* Harper and Row, New York (1958).

[7] W. Heisenberg, *The Physicist's Conception of Nature,* translated by A. J. Pomerans, Harcourt Brace, New York (1958).

[8] A. Einstein, Reply to Criticisms, in [5].

[9] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[10] R. B. Griffiths, J. Stat. Phys. **36**, 219 (1984).

[11] R. Omnes, J. Stat. Phys. **53**, 893 (1988); **53**, 933 (1988); **53**, 957 (1988).

[12] M. Gell-Mann and J. B. Hartle, Quantum Mechanics in the Light of Quantum Cosmology, in *Complexity, Entropy, and the Physics of Information,* W. Zurek, ed., Addison-Wesley, Reading (1990), p. 425; also in *Proceedings of the 3rd International Symposium on Quantum Mechanics in the Light of New Technology,* S. Kobayashi, H. Ezawa, Y. Murayama, and S. Nomura, eds., Physical Society of Japan (1990); Phys. Rev. D **47**, 3345 (1993).

[13] W. H. Zurek, Phys. Rev. D **26**, 1862 (1982); Physics Today **44**(10), 36 (1991); Physics Today **46**(4), 84 (1993).

[14] S. Goldstein and D. Page, Phys. Rev. Lett. **74**, 3715 (1995).

[15] A. M. Gleason, J. Math. and Mech. **6**, 885 (1957).

[16] S. Kochen and E. P. Specker, J. Math. and Mech. **17**, 59 (1967).

[17] J. S. Bell, Physics **1**, 195 (1964); reprinted in [1] and in [3].

[18] L. Hardy, Phys. Rev. Lett. **71**, 1665 (1993); S. Goldstein, Phys. Rev. Lett. **72**, 1951 (1994).

[19] M. Gell-Mann and J. B. Hartle, Alternative Decohering Histories in Quantum Mechanics, in the *Proceedings of the 25th International Conference on High Energy Physics, Singapore, August 2–8, 1990,* K. K. Phua and Y. Yamaguchi, eds., World Scientific, Singapore (1990).

[20] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).

[21] G. C. Ghirardi, Macroscopic Reality and the Dynamical Reduction Program, invited lecture to the Tenth International Congress of Logic, Methodology, and Philosophy of Science, Florence, August 1995, in *Structures and Norms in Science,* M. L. Dalla Chiara et al., eds., Kluwer Academic Publishers, Dordrecht (1997).

[22] K. Berndl, D. Dürr, S. Goldstein, G. Peruzzi, and N. Zanghì, Commun. Math. Phys. **173**, 647 (1995), quant-ph/9503013.

[23] D. Bohm, Phys. Rev. **85**, 166 (1952); **85**, 180 (1952).

[24] D. Dürr, S. Goldstein, and N. Zanghì, J. Stat. Phys. **67**, 843 (1992).

[25] J. S. Bell, Rev. Mod. Phys. **38**, 447 (1966); reprinted in [1] and in [3].

[26] E. P. Wigner, Interpretation of Quantum Mechanics, in [3].

[27] M. Born, Z. Phys. **38**, 803 (1926); English translation in *Wave Mechanics,* G. Ludwig, ed., Pergamon Press, Oxford (1968), p. 206.

[28] L. de Broglie, La Nouvelle Dynamique des Quanta, in [29].

[29] *Electrons et Photons: Rapports et Discussions du Cinquième Conseil de Physique tenu à Bruxelles du 24 au 29 Octobre 1927 sous les Auspices de l'Institut International de Physique Solvay,* Gauthier-Villars, Paris (1928).

[30] W. Pauli, in [29].

[31] D. Dürr, S. Goldstein, and N. Zanghì, Bohmian Mechanics as the Foundation of Quantum Mechanics, in *Bohmian Mechanics and Quantum Theory: An Appraisal*, J. Cushing, A. Fine, and S. Goldstein, eds., Kluwer Academic Publishers, Dordrecht (1996), quant-ph/9511016.

[32] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik,* Springer Verlag, Berlin (1932); English translation by R. T. Beyer, *Mathematical Foundations of Quantum Mechanics,* Princeton University Press, Princeton (1955).

[33] N. D. Mermin, Rev. Mod. Phys. **65**, 803 (1993).

[34] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics,* Volume I, Addison-Wesley, New York (1963).

[35] R. Feynman, *The Character of Physical Law,* The MIT Press, Cambridge (1967).

[36] C. Philippidis, C. Dewdney, and B. J. Hiley, Nuovo Cimento **52B**, 15 (1979).

Figure 2: Though he formulated its fundamental equation, Schrödinger was one of quantum theory's most acerbic critics.

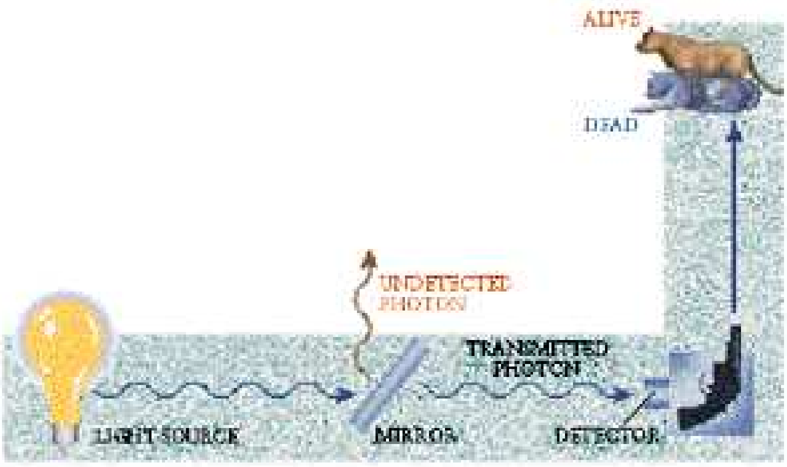

Figure 3: For Einstein, Schrödinger's cat, or the measurement problem, was no paradox. Rather, it merely demonstrated that the wave function does not provide a complete description of physical reality, for [8, page 671] "If we attempt [to work with] the interpretation that the quantum-mechanical description is to be understood as a complete description of the individual system, we are forced to the interpretation that the location of the mark on the strip [or the fact as to whether the cat is dead or alive] is nothing which belongs to the system *per se,* but that the existence of that location is essentially dependent upon the carrying out of an observation made on the registration-strip. Such an interpretation is certainly by no means absurd from a purely logical standpoint; yet there is hardly likely to be anyone who would be inclined to consider it seriously."

Figure 4: The leading figures of twentieth century physics, Einstein and Bohr engaged in a decades-long debate about the meaning and interpretation of quantum mechanics.

Figure 5: Over the past half century, Murray Gell-Mann has been one of the most sensible critics of orthodox quantum theory while Richard Feynman was one of its most sensible defenders.

Figure 6: For the past several decades, John Bell was the deepest thinker on the foundations of quantum mechanics. His analysis of nonlocality and hidden variables revitalized the field. Unfortunately, the implications of his work have been widely misunderstood.

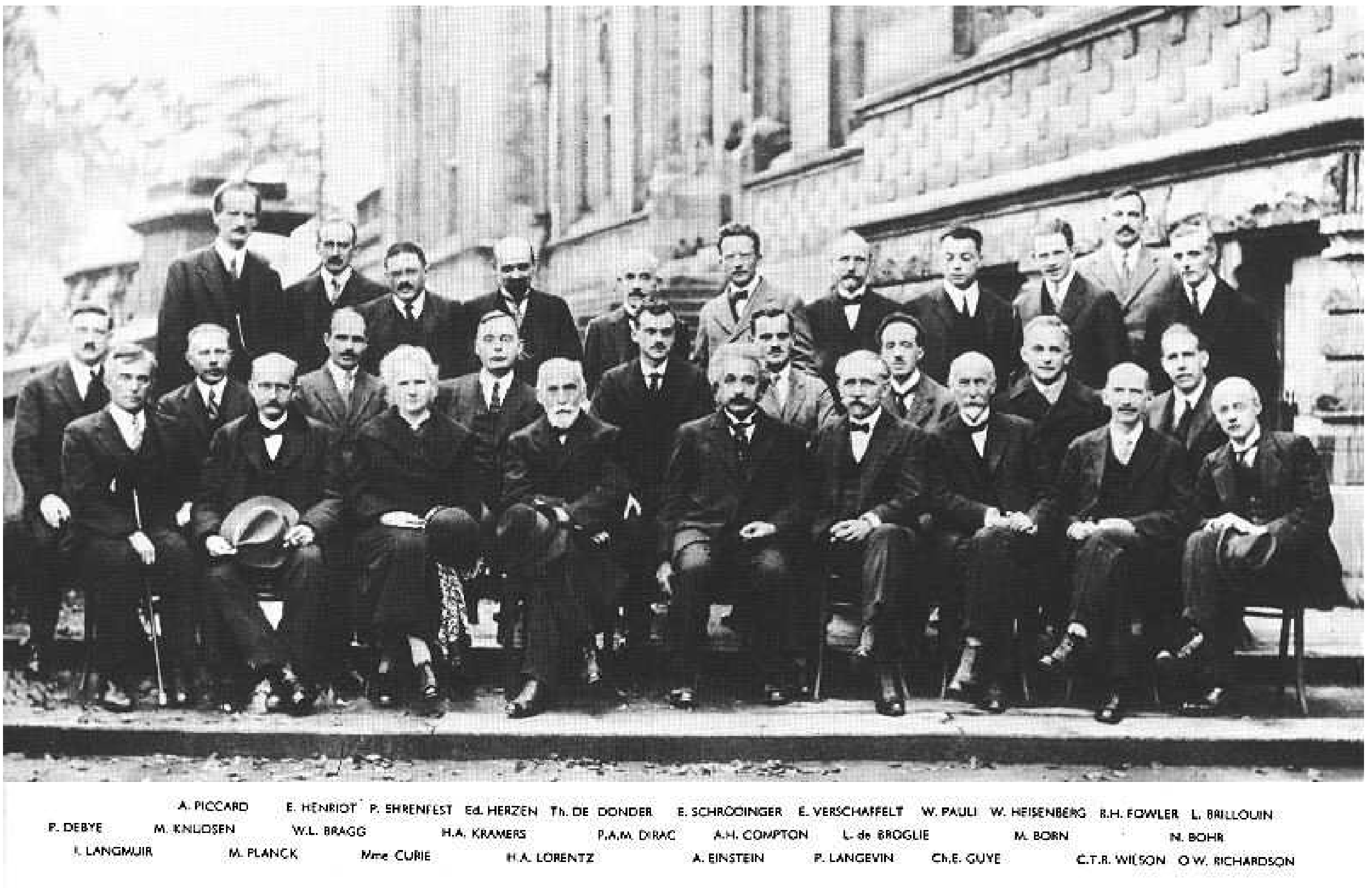

Figure 7: The participants of the Fifth Solvay Congress

Figure 8: Some of David Bohm's ideas about quantum mechanics and the nature of physical reality, for example regarding the implicate order, were rather speculative, but his deterministic version of quantum mechanics is quantum theory's most lucid and straightforward completion.