Quantum Spacetime without Observers: Ontological Clarity and the Conceptual Foundations of Quantum Gravity*

Sheldon Goldstein
Department of Mathematics, Rutgers University,
New Brunswick, NJ 08903, USA

Stefan Teufel
Mathematisches Institut der Universität München,
Theresienstraße 39, 80333 München, Germany

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Abstract

We explore the possibility of a Bohmian approach to the problem of finding a quantum theory incorporating gravitational phenomena. The major *conceptual* problems of canonical quantum gravity are the problem of time and the problem of diffeomorphism invariant observables. We find that these problems are artifacts of the subjectivity and vagueness inherent in the framework of orthodox quantum theory. When we insist upon ontological clarity—the distinguishing characteristic of a Bohmian approach—these conceptual problems vanish. We shall also discuss the implications of a Bohmian perspective for the significance of the wave function, concluding with unbridled speculation as to why the universe should be governed by laws so apparently bizarre as those of quantum mechanics.

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1 Introduction

> The term "3-geometry" makes sense as well in quantum geometrodynamics as in classical theory. So does superspace. But space-time does not. Give a 3-geometry, and give its time rate of change. That is enough, under typical circumstances to fix the whole time-evolution of the geometry; enough in other words, to determine the entire four-dimensional space-time geometry, provided one is considering the problem in the context of classical physics. In the real world of quantum physics, however, one cannot give both a dynamic variable and its time-rate of change. The principle of complementarity forbids. Given the precise 3-geometry at one instant, one cannot also know at that instant the time-rate of change of the 3-geometry. ...The uncertainty principle thus deprives one of any way whatsoever to predict, or even to give meaning to, "the deterministic classical history of space evolving in time." *No prediction of spacetime, therefore no meaning for spacetime,* is the verdict of the quantum principle.
> Misner, Thorne, Wheeler 1973

One of the few propositions about quantum gravity that most physicists in the field would agree upon, that our notion of space-time must, at best, be altered considerably in any theory conjoining the basic principles of quantum mechanics with those of general relativity, will be questioned in this article. We will argue, in fact, that most, if not all, of the conceptual problems in quantum gravity arise from the sort of thinking on display in the preceding quotation.

It is also widely agreed, almost 40 years after the first attempts to quantize general relativity, that there is still no single set of ideas on how to proceed, and certainly no physical theory successfully concluding this program. Rather, there are a great variety of approaches to quantum gravity; for a detailed overview, see, e.g., Rovelli [20]. While the different approaches to quantum gravity often have little in common, they all are intended ultimately to provide us with a consistent quantum theory agreeing in its predictions with general relativity in the appropriate physical domain. Although we will focus here on the conceptual problems faced by those approaches which amount to a canonical quantization of classical general relativity, the main lessons will apply to most of the other approaches as well.

This is because, as we shall argue, many of these difficulties arise from the subjectivity and the ontological vagueness inherent in the very framework of orthodox quantum theory, a framework taken for granted by almost all approaches to quantum gravity. We shall sketch how most, and perhaps all, of the conceptual problems of canonical quantum gravity vanish if we

insist upon formulating our cosmological theories in such a manner that it is reasonably clear what they are about—if we insist, that is, upon ontological clarity—and, at the same time, avoid any reference to such vague notions as measurement, observers, and observables.

The earliest approach, canonical quantum gravity, amounts to quantizing general relativity according to the usual rules of canonical quantization. However, to apply canonical quantization to general relativity, the latter must first be cast into canonical form. Since the quantization of the standard canonical formulation of general relativity, the Arnowitt, Deser, Misner formulation [1], has led to severe conceptual and technical difficulties, nonstandard choices of canonical variables, such as in the Ashtekar formulation [2] and in loop quantum gravity [21], have been used as starting points for quantization. While some of the technical problems have been resolved by these new ideas, the basic conceptual problems have not been addressed.

After the great empirical success of the standard model in particle physics, the hope arose that the gravitational interaction could also be incorporated in a similar model. The search for such a unified theory led to string theory, which apparently reproduces not only the standard model but also general relativity in a suitable low energy limit. However, since string theory is, after all, a quantum theory, it retains all the conceptual difficulties of quantum theory, and our criticisms and conclusions pertaining to quantum theory in general, in Sections 3 and 4 of this article, will apply to it as well. Nonetheless, our focus, again, will be on the canonical approaches, restricted for simplicity to pure gravity, ignoring matter.

This article is organized as follows: In Section 2 we will sketch the fundamental conceptual problems faced by most approaches to quantum gravity. The seemingly unrelated problems in the foundations of orthodox quantum theory will be touched upon in Section 3. Approaches to the resolution of these problems based upon the demand for ontological clarity will be discussed in Section 4, where we will focus on the simplest such approach, the de Broglie-Bohm theory or Bohmian mechanics. Our central point will be made in Section 5, where we indicate how the conceptual problems of canonical quantum gravity disappear when the main insights of the Bohmian approach to quantum theory are applied.

Finally, in Section 6, we will discuss how the status and significance of the wave function, in Bohmian mechanics as well as in orthodox quantum theory, is radically altered when we adopt a universal perspective. This altered status of the wave function, together with the very stringent symmetry demands so

central to general relativity, suggests the possibility—though by no means the inevitability—of finding an answer to the question, Why should the universe be governed by laws so apparently peculiar as those of quantum mechanics?

2 The conceptual problems of quantum gravity

In the canonical approach to quantum gravity one must first reformulate general relativity as a Hamiltonian dynamical system. This was done by ADM [1], using the 3-metric $g_{ij}(x^a)$ on a space-like hypersurface Σ as the configurational variable and the extrinsic curvature of the hypersurface as its conjugate momentum $\pi^{ij}(x^a)$[1]. The real time parameter of usual Hamiltonian systems is replaced by a "multi-fingered time" corresponding to arbitrary deformations $d\Sigma$ of the hypersurface. These deformations are split into two groups: those changing only the three dimensional coordinate system x^a on the hypersurface (with which, as part of what is meant by the hypersurface, it is assumed to be equipped) and deformations of the hypersurface along its normal vector field. While the changes of the canonical variables under both kind of deformations are generated by Hamiltonian functions on phase space, $H_i(g,\pi)$ for spatial diffeomorphisms and $H(g,\pi)$ for normal deformations, their changes under pure coordinate transformations on the hypersurfaces are dictated by their geometrical meaning. The dynamics of the theory is therefore determined by the Hamiltonian functions $H(g,\pi)$ generating changes under normal deformations of the hypersurface.

Denote by $N(x^a)$ the freely specifiable lapse function that determines how far, in terms of proper length, one moves the space-like hypersurface at the point $x = (x^a)$ along its normal vector: This distance is $N(x^a)d\tau$, where τ is a parameter labeling the successive hypersurfaces arrived at under the deformation (and defined by this equation). The infinitesimal changes of the canonical variables are then generated by the Hamiltonian H_N associated with N (an integral over Σ of the product of N with a Hamiltonian density $H(g,\pi;x^a)$):

[1] Actually the extrinsic curvature is given by $K_{ij} = G_{ijab}\pi^{ab}$ where G_{ijab} is the so called supermetric, which is itself a function of g_{ij}. This distinction is, however, not relevant to our discussion.

$$
\begin{aligned}
dg_{ij}(x^a) &= \frac{\delta H_N(g,\pi)}{\delta \pi^{ij}(x^a)} d\tau \\
d\pi^{ij}(x^a) &= -\frac{\delta H_N(g,\pi)}{\delta g_{ij}(x^a)} d\tau \,. \qquad (1)
\end{aligned}
$$

In what follows we shall denote by $H(g,\pi)$ the collection $\{H_N(g,\pi)\}$ of all such Hamiltonians (or, what comes pretty much to the same thing, the collection $\{H(g,\pi;x)\}$ for all points $x \in \Sigma$) and similarly for H_i.

It is important to stress that the theory can be formulated completely in terms of geometrical objects on a three dimensional manifold, with no a priori need to refer to its embedding into a space-time. A solution of (1) is a family of 3-metrics $g(\tau)$ that can be glued together to build up a 4-metric using the lapse function N (to determine the transverse geometry). In this way the space-time metric emerges dynamically when one evolves the canonical variables with respect to multi-fingered time.

However, the initial canonical data cannot be chosen arbitrarily, but must obey certain constraints: Only for initial conditions that lie in the submanifold of phase space on which $H_i(g,\pi)$ and $H(g,\pi)$ vanish do the solutions (space-time metrics $g_{\mu\nu}(x^\mu)$) also satisfy Einstein's equations. In fact, away from this so called constraint manifold the theory is not even well defined, at least not as a theory involving a multi-fingered time, since the solutions would depend on the special way we choose to evolve the space-like hypersurface, i.e., on the choice of $N(x^a)$, to build up space-time. Of course, a theory based on a single choice, for example $N(x^a) = 1$, would be well defined, at least formally.

By the same token, the invariance of the theory under space-time diffeomorphisms is no longer so obvious as in the formulation in terms of Einstein's equations: In the ADM formulation 4-diffeomorphism invariance amounts to the requirement that one ends up with the same space-time, up to coordinate transformations, regardless of which path in multi-fingered time is followed, i.e., which lapse function N, or τ-dependent sequence of lapse functions $N(\tau)$, is used. This says that for the space-time built up from any particular choice of multi-fingered time, the dynamical equations (1) will be satisfied for *any* foliation of the resulting space-time into space-like hypersurfaces—using in (1) the lapse function $N(\tau)$ associated with that foliation—and not just for the foliation associated with that particular choice.

Formally, it is now straightforward to quantize this constrained Hamiltonian theory using Dirac's rules for the quantization of constrained systems [8]. First one must replace the canonical variables g_{ij} and π^{ij} by operators $\hat{g}_{ij}$ and $\hat{\pi}^{ij} = -i\frac{\delta}{\delta g_{ij}}$ satisfying the canonical commutation relations.[2] One then formally inserts these into the Hamiltonian functions $H(g,\pi)$ and $H_i(g,\pi)$ of the classical theory to obtain operators $\widehat{H}(\hat{g},\hat{\pi})$ and $\widehat{H}_i(\hat{g},\hat{\pi})$ acting on functionals $\Psi(g)$ on the configuration space of 3-metrics. Since the Hamiltonians were constrained in the classical theory one demands that the corresponding operators annihilate the physical states in the corresponding quantum theory:

$$\widehat{H}\Psi = 0 \tag{2}$$

$$\widehat{H}_i\Psi = 0\,. \tag{3}$$

Equation (3) has a simple meaning, namely that $\Psi(g)$ be invariant under 3-diffeomorphisms (coordinate changes on the 3-manifold), so that it depends on the 3-metric g only through the 3-geometry. However, the interpretation of the *Wheeler-DeWitt equation* (2) is not at all clear.

Before discussing the several problems which arise in attempts to give a physical meaning to the approach just described, a few remarks are in order: While we have omitted many technical details and problems from our schematic description of the "Dirac constraint quantization" of gravity, these problems either do not concern, or are consequences of, the main conceptual problems of canonical quantum gravity. Other approaches, such as the canonical quantization of the Ashtekar formulation of classical general relativity and its further development into loop quantum gravity, resolve some of the technical problems faced by canonical quantization in the metric representation, but leave the main conceptual problems untouched.

Suppose now that we have found a solution $\Psi(g)$ to equations (2) and (3). What physical predictions would be implied? In orthodox quantum theory a solution Ψ_t of the time-dependent Schrödinger equation provides us with a time-dependent probability distribution $|\Psi_t|^2$, as well as with the absolute square of other time-dependent probability amplitudes. The measurement problem and the like aside, the physical meaning of these is reasonably clear: they are probabilities for the results of the measurement of the configuration or of other observables. But any attempt to interpret canonical quantum gravity along orthodox lines immediately faces the following problems:

[2] We choose units in which $\hbar$ and c are 1.

- **The problem of time**: In canonical quantum gravity there is no time-dependent Schrödinger equation; it was replaced by the time-independent Wheeler-DeWitt equation. The Hamiltonians—the generators of multi-fingered-time evolution in the classical case—annihilate the state vector and therefore cease to generate any evolution at all. The theory provides us with only a timeless wave function on the configuration space of 3-metrics, i.e., on the possible configurations of space, not of space-time. But how can a theory that provides us (at best) with a single fixed probability distribution for configurations of space ever be able to describe the always changing world in which we live? This, in a nutshell, is the problem of time in canonical quantum gravity.

- **The problem of 4-diffeomorphism invariance**: The fundamental symmetry at the heart of general relativity is its invariance under general coordinate transformations of space-time. It is important to stress that almost any theory can be formulated in such a 4-diffeomorphism invariant manner by adding further structure to the theory (e.g., a preferred foliation of space-time as a dynamical object). General relativity has what is sometimes called serious diffeomorphism-invariance, meaning that it involves no space-time structure beyond the 4-metric and, in particular, singles out no special foliation of space-time. In canonical quantum gravity, while the invariance under coordinate transformations of space is retained, it is not at all clear what 4-diffeomorphism invariance could possibly mean. Therefore the basic symmetry, and arguably the essence, of general relativity seems to be lost in quantization.

- **The problem of "no outside observer":** One of the most fascinating applications of quantum gravity is to quantum cosmology. Orthodox quantum theory attains physical meaning only via its predictions about the statistics of outcomes of measurements of observables, performed by observers that are not part of the system under consideration, and seems to make no clear physical statements about the behavior of a closed system, not under observation. The quantum formalism concerns the interplay between—and requires for its very meaning—two kinds of objects: a quantum system and a more or less classical apparatus. It is hardly imaginable how one could make any sense out of this formalism for quantum cosmology, for which the system of interest

is the whole universe, a closed system if there ever was one.

- **The problem of diffeomorphism invariant observables:** Even if we pretend for the moment that we are able to give meaning to the quantum formalism without referring to an observer located outside of the universe, we encounter a more subtle difficulty. Classical general relativity is fundamentally diffeomorphism invariant. It is only the space-time geometry, not the 4-metric nor the identity of the individual points in the space-time manifold, that has physical significance. Therefore the physical observables in general relativity should be independent of special coordinate systems; they must be invariant under 4-diffeomorphisms, which are in effect generated by the Hamiltonians H and H_i. Since the quantum observables are constructed, via quantization, from the classical ones, it would seem that they must commute with the Hamiltonians $\widehat{H}$ and $\widehat{H}_i$. But such diffeomorphism invariant quantum observables are extremely hard to come by, and there are certainly far too few of them to even begin to account for the bewildering variety of our experience which it is the purpose of quantum theory to explain. (For a discussion of the question of existence of diffeomorphism invariant observables, see Kuchař [17].)

These conceptual problems, and the attempts to solve them, have lead to a variety of technical problems that are discussed in much detail in, e.g., Kuchař [17], [18] and Isham [15]. However, since we are not aware of any orthodox proposals successfully resolving the conceptual problems, we shall not discuss such details here. Rather, we shall proceed in the opposite direction, toward their common cause, and argue that they originate in a deficiency shared with, and inherited from, orthodox quantum mechanics: the lack of a coherent ontology.

Regarding the first two problems of canonical quantum gravity, it is not hard to discern their origin: the theory is concerned only with configurations of and on space, the notion of a space-time having entirely disappeared. It is true that even with classical general relativity, Newton's external absolute time is abandoned. But a notion of time, for an observer located somewhere in space-time and employing a coordinate system of his convenience, is retained, emerging from space-time. The problem of time in canonical quantum gravity is a direct consequence of the fact that in an orthodox quantum theory for space-time itself we must insist on its nonexistence (compare the

quote at the beginning of this article). Similarly, the problem of diffeomorphism invariance, or, better, the problem of not even being able to address this question properly, is an immediate consequence of having no notion of space-time in orthodox quantum gravity.

3 The basic problem of orthodox quantum theory: the lack of a coherent ontology

Despite its extraordinary predictive successes, quantum theory has, since its inception some seventy-five years ago, been plagued by severe conceptual difficulties. The most widely cited of these is the measurement problem, best known as the paradox of Schrödinger's cat. For many physicists the measurement problem is, in fact, not *a* but *the* conceptual difficulty of quantum theory.

In orthodox quantum theory the wave function of a physical system is regarded as providing its complete description. But when we analyze the process of measurement itself in quantum mechanical terms, we find that the after-measurement wave function for system and apparatus arising from Schrödinger's equation for the composite system typically involves a superposition over terms corresponding to what we would like to regard as the various possible results of the measurement—e.g., different pointer orientations. Since it seems rather important that the actual result of the measurement be a part of the description of the after-measurement situation, it is difficult to believe that the wave function alone provides the complete description of that situation.

The usual collapse postulate for quantum measurement solves this problem for all practical purposes, but only at the very steep price of the introduction of an *observer* or *classical measurement apparatus* as an irreducible, unanalyzable element of the theory. This leads to a variety of further problems. The unobserved physical reality becomes drastically different from the observed, even on the macroscopic level of daily life. Even worse, with the introduction at a fundamental level of such vague notions as classical measurement apparatus, the physical theory itself becomes unprofessionally vague and ill defined. The notions of observation and measurement can hardly be captured in a manner appropriate to the standards of rigor and clarity that should be demanded of a fundamental physical theory. And in

quantum cosmology the notion of an external observer is of course entirely obscure.

The collapse postulate is, in effect, an unsuccessful attempt to evade the measurement problem without taking seriously its obvious implication: that the wave function does not provide a complete description of physical reality. If we do accept this conclusion, we must naturally inquire about the nature of the more complete description with which a less problematical formulation of quantum theory should be concerned. We must ask, which theoretical entities, in addition to the wave function, might the theory describe? What mathematical objects and structures represent entities that, according to the theory, simply *are*, regardless of whether or not they are observed? We must ask, in other words, about the *primitive ontology* of the theory, what the theory is fundamentally about (see Goldstein [11]). And when we know what the theory is really about, measurement and observation become secondary phenomenological concepts that, like anything else in a world governed by the theory, can be analyzed in terms of the behavior of its primitive ontology.

By far the simplest possibility for the primitive ontology is that of particles described by their positions. The corresponding theory, for non-relativistic particles, is Bohmian mechanics.

4 Bohmian mechanics

According to Bohmian mechanics the complete description of an n-particle system is provided by its wave function Ψ together with its configuration $Q = (\mathbf{Q}_1, \ldots, \mathbf{Q}_n)$, where the $\mathbf{Q}_k$ are the positions of its particles. The wave function, which evolves according to Schrödinger's equation, choreographs the motion of the particles: these evolve—in the simplest manner possible—according to a first-order ordinary differential equation

$$\frac{dQ}{dt} = v^{\Psi}(Q)$$

whose right hand side, a velocity vector field on configuration space, is generated by the wave function. Considerations of simplicity and space-time symmetry—Galilean and time-reversal invariance—then determine the form of v^{Ψ}, yielding the defining (evolution) equations of Bohmian mechanics (for spinless particles):

$$\frac{d\mathbf{Q}_k}{dt} = \mathbf{v}_k^{\Psi}(\mathbf{Q}_1, \ldots, \mathbf{Q}_n) = \frac{\hbar}{m_k} \mathrm{Im} \frac{\nabla_{\mathbf{q}_k} \Psi}{\Psi}(\mathbf{Q}_1, \ldots, \mathbf{Q}_n) \tag{4}$$

and

$$i\hbar\frac{\partial\Psi}{\partial t} = \widehat{H}\Psi \tag{5}$$

where $\widehat{H}$ is the usual Schrödinger Hamiltonian, containing as parameters the masses $m_1, \ldots, m_n$ of the particles as well as the potential energy function V of the system. For an n-particle universe, these two equations form a complete specification of the theory. There is no need, and indeed no room, for any further axioms, describing either the behavior of other "observables" or the effects of "measurement."

Bohmian mechanics is the most naively obvious embedding imaginable of Schrödinger's equation into a completely coherent physical theory! If one didn't already know better, one would naturally conclude that it can't "work," i.e., that it can't account for quantum phenomena. After all, if something so obvious and, indeed, so trivial works, great physicists—so it would seem—would never have insisted, as they have and as they continue to do, that quantum theory demands radical epistemological and metaphysical innovations.

Moreover, it is hard to avoid wondering how Bohmian mechanics *could* have much to do with quantum theory? Where is quantum randomness in this deterministic theory? Where is quantum uncertainty? Where are operators as observables and all the rest?

Be that as it may, Bohmian mechanics is certainly *a* theory. It describes a world in which particles participate in a highly non-Newtonian motion, and it would do so even if this motion had absolutely nothing to do with quantum mechanics.

It turns out, however, as a surprising consequence of the equations (4) and (5), that when a system has wave function Ψ, its configuration is typically random, with probability density ρ given by $\rho = |\Psi|^2$, the *quantum equilibrium* distribution. In other words, it turns out that systems are somehow typically in quantum equilibrium. Moreover, this conclusion comes together with the clarification of what precisely this means, and also implies that a Bohmian universe embodies an absolute uncertainty which can itself be regarded as the origin of the uncertainty principle. We shall not go into these matters here, since we have discussed them at length elsewhere (Dürr, Goldstein and Zanghì [9]). We note, however, that nowadays, with chaos theory and nonlinear dynamics so fashionable, it is not generally regarded as terribly astonishing for an appearance of randomness to emerge from a deterministic dynamical system.

It also turns out that the entire quantum formalism, operators as observables and all the rest, is a consequence of Bohmian mechanics, emerging from an analysis of idealized measurement-like situations (for details, see Daumer et al. [6] and [7]; see also Bohm [4]). There is no measurement problem in Bohmian mechanics because the complete description of the after-measurement situation includes, in addition to the wave function, the definite configuration of the system and apparatus. While the wave function may still be a superposition of states corresponding to macroscopically different possible outcomes, the actual configuration singles out the outcome that has occurred.

Why have we elaborated in such detail on non-relativistic quantum mechanics and Bohmian mechanics if our main concern here is with quantum gravity? Because there are two important lessons to be learned from a Bohmian perspective on quantum theory. First of all, the existence of Bohmian mechanics demonstrates that the characteristic features of quantum theory, usually viewed as fundamental—intrinsic randomness, operators as observables, non-commutativity, and uncertainty—need play no role whatsoever in the formulation of a quantum theory, naturally emerging instead, as a consequence of the theory, in special measurement-like situations. Therefore we should perhaps not be too surprised when approaches to quantum gravity that regard these features as fundamental encounter fundamental conceptual difficulties. Second, the main point of our paper is made transparent in the simple example of Bohmian mechanics. If we base our theory on a coherent ontology, the conceptual problems may disappear, and, what may be even more important, a genuine understanding of the features that have seemed most puzzling might be achieved.

We shall now turn to what one might call a Bohmian approach to quantum gravity.

5 Bohmian quantum gravity

The transition from quantum mechanics to Bohmian mechanics is very simple, if not trivial: one simply incorporates the actual configuration into the theory as the basic variable, and stipulates that this evolve in a natural way, suggested by symmetry and by Schrödinger's equation. The velocity field

v^{Ψ_t} is, in fact, related to the quantum probability current j^{Ψ_t} by

$$v^{\Psi_t} = \frac{j^{\Psi_t}}{|\Psi_t|^2}\,,$$

suggesting, since $\rho^{\Psi} = |\Psi|^2$ satisfies the continuity equation with $j^{\Psi_t} = \rho^{\Psi_t} v^{\Psi_t}$, that the empirical predictions of Bohmian mechanics, for positions and ultimately, in fact, for other "observables" as well, agree with those of quantum mechanics (as in fact they do; see Dürr et al. [9]).

Formally, one can follow exactly the same procedure in canonical quantum gravity, where the configuration space is the space of (positive-definite) 3-metrics (on an appropriate fixed manifold). The basic variable in Bohmian quantum gravity is therefore the 3-metric g (representing the geometry on a space-like hypersurface of the space-time to be generated by the dynamics) and its change under (what will become) normal deformations is given by a vector field on configuration space generated by the wave function $\Psi(g)$. Considerations analogous to those for non-relativistic particles lead to the following form for the Bohmian equation of motion:

$$dg_{ij}(x^a) = G_{ijab}(x^a)\mathrm{Im}\left(\Psi(g)^{-1}\frac{\delta\Psi(g)}{\delta g_{ab}(x^a)}\right) N(x^a)\,d\tau\,. \tag{6}$$

The wave function $\Psi(g)$ is a solution of the timeless Wheeler-DeWitt equation (2) and therefore does not evolve. But the vector field on the right hand side of (6) that it generates is typically nonvanishing if $\Psi(g)$ is complex, leading to a nontrivial evolution $g(\tau)$ of the 3-metric. Suitably gluing together the 3-metrics $g(\tau)$, we obtain a space-time (see the paragraph after equation (1)). Interpretations of canonical quantum gravity along these lines have been proposed by, e.g., Holland [14] and discussed, e.g., by Shtanov [22]. Minisuperspace Bohmian cosmologies have been considered by Kowalski-Glikman and Vink [16], Squires [23] and Callender and Weingard [5].

However, there is a crucial point which is often overlooked or, at least, not made sufficiently clear in the literature. A space-time generated by a solution of (2) via (6) will in general depend on the choice of lapse function N (or $N(\tau)$). Thus the theory is not well defined as so far formulated. There are essentially two ways to complete the theory. Either one chooses a special lapse function N, e.g., $N = 1$, or one employs only special solutions Ψ of (2), those yielding a vector field that generates an N-independent space-time. In the first case, with special N but general solution Ψ of (2), the general

covariance of the theory will typically be broken, the theory incorporating a special foliation (see the paragraph before the one containing equation (2)). The possible existence of special solutions giving rise to a covariant dynamics will be discussed in more detail elsewhere (Goldstein and Teufel, [12]), and will be touched upon towards the end of Section 6. However, most the following discussion, especially in the first part of Section 6, does not depend upon whether or not the theory incorporates a special foliation.

Let us now examine the impact of the Bohmian formulation of canonical quantum gravity on the basic conceptual problems of orthodox canonical quantum gravity. Since a solution to the equations of Bohmian quantum gravity defines a space-time, the problem of time is resolved in the most satisfactory way: Time plays exactly the same role as in classical general relativity; there is no need whatsoever for an external absolute time, which has seemed so essential to orthodox quantum theory. The problem of diffeomorphism-invariance is ameliorated, in that in this formulation it is at least clear what diffeomorphism-invariance means. But, as explained above, general covariance can be expected at most for special solutions of (2). If it should turn out, however, that we must abandon general covariance on the fundamental level by introducing a special foliation of space-time, it may still be possible to retain it on the observational level (see, e.g., Münch-Berndl et al. [19], where it is also argued, however, that a special, dynamical, foliation of space-time need not be regarded as incompatible with serious covariance).

A short answer to the problems connected with the role of observers and observables is this: There can be no such problems in the Bohmian formulation of canonical quantum gravity since observers and observables play no role in this formulation. But this is perhaps too short. What, after all, is wrong with the observation that, since individual space-time points have no physical meaning, physically significant quantities must correspond to diffeomorphism-invariant observables, of which there are far too few to describe very much of what we most care about?

The basic answer, we believe, is this: We ourselves are not—or, at least, need not be[3]—diffeomorphism invariant: Most physical questions of relevance to us are not formulated in a diffeomorphism invariant manner because, naturally enough, they refer to our own special location in space-time.

[3]In some models of quantum cosmology, e.g., those permitting the definition of a global time function, it may well be possible to pick ourselves out in a diffeomorphism-invariant manner.

Nonetheless, we know very well what they mean—we know, e.g., what it means to ask where and when something happens with respect to our own point of view. Such questions can be addressed, in fact because of diffeomorphism invariance, by taking into account the details of our environment and asking about the local predictions of the theory conditioned on such an environment, past and present.

The observer who sets the frame of reference for his physical predictions is part of and located inside the system—the universe. In classical general relativity this is not at all problematical, since that theory provides us with a coherent ontology, a potentially complete description of space-time and, if we wish, a description taking into account our special point of view in the universe. But once the step to quantum theory is taken, the coherent space-time ontology is replaced by an incoherent "ontology" of quantum observables. In orthodox quantum theory this problem can be talked away by introducing an outside observer actually serving two purposes: the observer sets the frame of reference with respect to which the predictions are to be understood, a totally legitimate and sensible purpose. But of course the main reason for the focus on observers in quantum theory is that it is only with respect to them that the intrinsically incoherent quantum description of the system under observation can be given any meaning. In quantum cosmology, however, no outside observer is at hand, neither for setting a frame of reference nor for transforming the incoherent quantum picture into a coherent one.

In Bohmian quantum gravity, again, both problems disappear. Since we have a coherent description of the system itself, in this case the universe, there is no need for an outside observer in order to give meaning to the theory. Nor do we have to worry about the diffeomorphism invariance of observables, since we are free to refer to observers who are themselves part of the system.

There is, however, an important aspect of the problem of time that we have not yet addressed. From a Bohmian perspective, as we have seen, a time-dependent wave function, satisfying Schrödinger's equation, is by no means necessary to understand the possibility of what we call change. Nonetheless, a great deal of physics is, in fact, described by such time-dependent wave functions. We shall see in the next section how these also naturally emerge from the structure of Bohmian quantum gravity, which fundamentally has only a timeless universal wave function.

6 A universal Bohmian theory

When Bohmian mechanics is viewed from a universal perspective, the status of the wave function is dramatically altered. To appreciate what we have in mind here, it might help to consider two very common objections to Bohmian mechanics.

Bohmian mechanics violates the action-reaction principle that is central to all of modern physics, both classical and (non-Bohmian) quantum: In Bohmian mechanics there is no back-action of the configuration upon the wave function, which evolves, autonomously, according to Schrödinger's equation. And the wave function, which is part of the state description of—and hence presumably part of the reality comprising—a Bohmian universe, is not the usual sort of physical field on physical space (like the electromagnetic field) to which we are accustomed, but rather a field on the abstract space of all possible configurations, a space of enormous dimension, a space constructed, it would seem, by physicists as a matter of convenience.

It should be clear by now what, from a universal viewpoint, the answer to these objections must be: As first suggested by Dürr et al. [10], the wave function Ψ of the universe should be regarded as a representation, not of substantial physical reality, but of physical law. In a universal Bohmian theory Ψ should be a functional of the configurations of all elements of physical reality: geometry, particle positions, field or string configurations, or whatever primitive ontology turns out to describe nature best. As in the case of pure quantum gravity, Ψ should be a (special) solution of some fundamental equation (such as the Wheeler-DeWitt equation (2) with additional terms for particles, fields, etc.). Such a universal wave function would be static—a wave function whose timelessness constitutes the problem of time in canonical quantum gravity—and, insofar as our universe is concerned, unique. But this doesn't mean, as we have already seen, that the world it describes would be static and timeless. No longer part of the state description, the universal wave function Ψ provides a representation of dynamical law, via the vector field on configuration space that it defines. As such, the wave function plays a role analogous to that of the Hamiltonian function $H = H(Q, P) \equiv H(\xi)$ in classical mechanics—a function on phase space, a space even more abstract than configuration space. In fact, the wave function and the Hamiltonian function generate motions in pretty much the same way

$$\frac{d\xi}{dt} = \mathrm{Der}H \longleftrightarrow \frac{dQ}{dt} = \mathrm{Der}(\log \Psi)\,,$$

with Der a derivation. And few would be tempted to regard the Hamiltonian function H as a real physical field, or expect any back-action of particle configurations on this Hamiltonian function.

Once we recognize that the role of the wave function is thus nomological, two important questions naturally arise: Why and how should a formalism involving time-dependent wave functions obeying Schrödinger's equation emerge from a theory involving a fixed timeless universal wave function? And which principle singles out the special unique wave function Ψ that governs the motion in our universe? Our answers to these questions are somewhat speculative. But they do provide further insight into the role of the wave function in quantum mechanics and might even explain why, in fact, our world is quantum mechanical.

In order to understand the emergence of a time-dependent wave function, we must ask the right question, which is this: Is it ever possible to find a simple effective theory governing the behavior of suitable subsystems of a Bohmian universe? Suppose, then, that the configuration of the universe has a decomposition of the form $q = (x, y)$, where x describes the degrees of freedom with which we are somehow most directly concerned (defining the *subsystem*, the "x-system") and y describes the remaining degrees of freedom (the subsystem's *environment*, the "y-system"). For example, x might be the configuration of all the degrees of freedom governed by standard quantum field theory, describing the fermionic matter fields as well as the bosonic force fields, while y refers to the gravitational degrees of freedom. Suppose further that we have, corresponding to this decomposition, a solution $Q(\tau) = (X(\tau), Y(\tau))$ of the appropriate (yet to be defined) extension of (6), where the real continuous parameter τ labels the slices in a suitable foliation of space-time.

Focus now on the *conditional wave function*

$$\psi_\tau(x) = \Psi(x, Y(\tau)) \tag{7}$$

of the subsystem, governing its motion, and ask whether $\psi_\tau(x)$ could be—and might under suitable conditions be expected to be—governed by a simple law that does not refer directly to its environment. (The conditional wave function of the x-system should be regarded as defined only up to a factor that does not depend upon x.)

Suppose that Ψ satisfies an equation of the form (2), with $\widehat{H} = \{\widehat{H}_N\}$. Suppose further that for y in some "y-region" of configuration space and for

some choice of lapse function N we have that $\widehat{H}_N \simeq \widehat{H}_N^{(x)} + \widehat{H}_N^{(y)}$ and can write

$$\begin{aligned}\Psi(x,y) &= e^{-i\tau\widehat{H}_N}\Psi(x,y) \simeq e^{-i\tau\widehat{H}_N}\sum_\alpha \psi_0^\alpha(x)\phi_0^\alpha(y) \\ &\simeq \sum_\alpha \left(e^{-i\tau\widehat{H}_N^{(x)}}\psi_0^\alpha(x)\right)\left(e^{-i\tau\widehat{H}_N^{(y)}}\phi_0^\alpha(y)\right) \\ &=: \sum_\alpha \psi_\tau^\alpha(x)\phi_\tau^\alpha(y) \qquad (8)\end{aligned}$$

where the ϕ_0^α are "narrow disjoint wave packets" and remain approximately so as long as τ is not too large. Suppose (as would be the case for Bohmian mechanics) that the motion is such that if the configuration $Y(0)$ lies in the support of one $\phi_0^{\alpha'}$, then $Y(\tau)$ will keep up with $\phi_\tau^{\alpha'}$ as long as the above conditions are satisfied. It then follows from (8) that for the conditional wave function of the subsystem we have

$$\psi_\tau(x) \approx \psi_\tau^{\alpha'},$$

and it thus approximately satisfies the time-dependent Schrödinger equation

$$i\frac{\partial\psi}{\partial\tau} = \widehat{H}_N^{(x)}\psi\,. \qquad (9)$$

(In the case of (an extension of) Bohmian quantum gravity with preferred foliation, this foliation must correspond to the lapse function N in (8).)

We may allow here for an interaction $\widehat{W}_N(x,y)$ between the subsystem and its environment in the Hamiltonian in (8), provided that the influence of the x-system on the y-system is negligible. In this case we can replace $\widehat{H}_N^{(x)}$ in (8) and (9) by $\widehat{H}_N^{(x)}(Y(\tau)) \equiv \widehat{H}_N^{(x)} + \widehat{W}_N(x, Y(\tau))$, since the wave packets $\phi^\alpha(y)$ are assumed to be narrow. Think, for the simplest example, of the case in which the y-system is the gravitational field and the x-system consists of very light particles.

Now one physical situation (which can be regarded as corresponding to a region of configuration space) in which (8), and hence the Schrödinger evolution (9), should obtain is when the y-system behaves semiclassically: In the semiclassical regime, one expects an initial collection of narrow and approximately disjoint wave packets $\phi_0^\alpha(y)$ to remain so under their (approximately classical) evolution.

As a matter of fact, the emergence of Schrödinger's equation in the semiclassical regime for gravity can be justified in a more systematic way, using

perturbation theory, by expanding Ψ in powers of the gravitational constant κ. Then for a "semiclassical wave function" Ψ, the phase S of Ψ, to leading order, κ^{-1}, depends only on the 3-metric and obeys the classical Einstein-Hamilton-Jacobi equation, so that the metric evolves approximately classically, with the conditional wave function for the matter degrees of freedom satisfying, to leading (zeroth) order, Schrödinger's equation for, say, quantum field theory on a given evolving background. The relevant analysis was done by Banks [3] for canonical quantum gravity, but the significance of that analysis is rather obscure from an orthodox perspective:

The semiclassical limit has been proposed as a solution to the problem of time in quantum gravity, and as such has been severely criticized by Kuchař [17], who concludes his critique by observing that "the semiclassical interpretation does not solve the standard problems of time. It merely obscures them by the approximation procedure and, along the way, creates more problems." Perhaps the main difficulty is that, within the orthodox framework, the classical evolution of the metric is not really an approximation at all. Rather, it is put in by hand, and can in no way be justified on the basis of an entirely quantum mechanical treatment, even as an approximation. This is in stark contrast with the status of the semiclassical approximation within a Bohmian framework, for which there is no problem of time. In this approach, the classical evolution of the metric is indeed merely an approximation to its exact evolution, corresponding to the exact phase of the wave function (i.e., to equation (6)). To the extent that this approximation is valid, the appropriate conclusions can be drawn, but the theory makes sense, and suffers from no conceptual problems, even when the approximation is not valid.

Now to our second question. Suppose that we demand of a universal dynamics that it be first-order for the variables describing the primitive ontology (the simplest possibility for a dynamics) and covariant—involving no preferred foliation, no special choice of lapse function N, in its formulation. This places a very strong constraint on the vector field defining the law of motion—and on the universal wave function, should this motion be generated by a wave function. The set of wave functions satisfying this constraint should be very small, far smaller than the set of wave functions we normally consider as possible initial states for a quantum system. However, according to our conception of the wave function as nomological, this very fact might well be a distinct virtue.

We have begun to investigate the possibility of a first-order covariant geometrodynamics in connection with Bohmian quantum gravity, and have

found that the constraint for general covariance is captured by the Dirac algebra (see also Hojman, Kuchař and Teitelboim [13]), which expresses the relation between successive infinitesimal deformations of hypersurfaces, taken in different orders: We have shown (see Goldstein and Teufel [12]) that defining a representation of the Dirac algebra is more or less the necessary and sufficient condition for a vector field on the space of 3-metrics to yield a generally covariant dynamics, generating a 4-geometry involving no dynamically distinguished hypersurfaces.

This work is very much in its infancy. In addition to the problem of finding a mathematically rigorous proof of the result just mentioned, there remains the difficult question of the possible representations of the Dirac algebra, both for pure gravity and for gravity plus matter. For pure gravity it seems that a first-order generally covariant geometrodynamics is achievable, but only with vector fields that generate classical 4-geometries—solutions of the Einstein equations with a possible cosmological constant. How this situation might be affected by the inclusion of matter is not easy to say.

Even a negative result—to the effect that a generally covariant Bohmian theory must involve additional space-time structure—would be illuminating. A positive result—to the effect that a first-order dynamics, for geometry plus matter, that does not invoke additional space-time structure can be generally covariant more or less only when the vector field defining this dynamics arises from an essentially unique wave function of the universe that happens to satisfy an equation like the Wheeler-DeWitt equation (and from which a time-dependent Schrödinger equation emerges, in the manner we've described, as part of a phenomenological description governing the behavior of appropriate subsystems)—would be profound. For then we would know, not just what quantum mechanics is, but why it is.

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