1
Bell on Bohm

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1.1 Prologue

A memorial conference for John Bell, open to the public, was held at Rutgers University around 20 years ago. I gave there a short talk on Bell's views about David Bohm's "hidden variables" formulation of quantum mechanics, a version of quantum mechanics often called the de Broglie-Bohm theory or Bohmian mechanics. This theory was in fact discovered by Louis de Broglie in 1927. In 1952, it was rediscovered and developed by Bohm, who was the first to appreciate its connection to the predictions of standard quantum mechanics. I did not publish the talk.

I have decided that it would be appropriate to publish a very lightly edited version of it here, in this volume devoted to the work of Bell on the foundation of quantum mechanics. One reason for doing so is that the connection between Bell and Bohm continues to be somewhat underplayed, with the strength of his advocacy of Bohmian mechanics not properly appreciated. For example, about half of the papers in Bell's collected works on the foundations of quantum mechanics deal with Bohmian mechanics. But in his fine introduction to the revised edition of this great book (Bell, 2004), Alain Aspect mentions this theory in only a single sentence, and parenthetically at that.

For several decades after Bell proved his nonlocality theorem, based in part on Bell's inequality, it was widely claimed that Bell had shown that hidden variables—and Bohmian mechanics in particular—were impossible, that they were incompatible with the predictions of quantum mechanics. For example, the great physicist Eugene Wigner, who, unlike most of his contemporaries, was profoundly concerned with the conceptual foundations

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of quantum mechanics and usually wrote on the subject with great clarity and insight, has written that

> This [hidden variables] is an interesting idea and even though few of us were ready to accept it, it must be admitted that the truly telling argument against it was produced as late as 1965, by J. S. Bell. ... This appears to give a convincing argument against the hidden variables theory. (Wigner, 1983, p. 53)

More recently, we find Stephen Hawking writing that

> Einstein's view was what would now be called a hidden variable theory. Hidden variable theories might seem to be the most obvious way to incorporate the Uncertainty Principle into physics. They form the basis of the mental picture of the universe, held by many scientists, and almost all philosophers of science. But these hidden variable theories are wrong. The British physicist, John Bell, who died recently, devised an experimental test that would distinguish hidden variable theories. When the experiment was carried out carefully, the results were inconsistent with hidden variables. (Hawking, 1999)

So let's look at what Bell actually thought about the matter. Here is the talk.

1.2 Introduction

John Stewart Bell is best known for his discovery of the theorem that bears his name. This theorem establishes the impossibility of any explanation of quantum phenomena in terms of what are called local hidden variables.[2] And since one might well imagine that any account in terms of *nonlocal* hidden variables would have to be artificial—cooked up just to do the job—and generally unacceptable, Bell's theorem is widely regarded as precluding any hidden variable account worthy of our consideration. (As far as the meaning of a "hidden variable account" is concerned, for now let me just say, somewhat imprecisely, that a hidden variable formulation of quantum theory would eliminate quantum craziness while retaining the quantum predictions.) In other words, Bell's theorem is widely used to support the proposition that quantum phenomena demand radical epistemological and metaphysical innovations—precisely what hidden variables promise to avoid.

Now Bell wrote much and lectured much about his theorem and its implications. But he wrote and lectured as much, if not more, concerning the virtues of what is the most famous of all hidden variable theories, that of

[2] I now regret having written that sentence. While not wrong, it is nonetheless misleading. What Bell established was the impossibility of any local account of quantum phenomena (that does not involve many worlds), not just the impossibility of an explanation in terms of local hidden variables. That this is what Bell himself thought is clear, for example, from the quotations of section 1.8.

David Bohm. The question thus naturally arises, why would Bell spend so much time and effort expounding upon a theory of just the sort that he himself had shown to be, if not impossible, unworthy of consideration?

Indeed, some physicists have spoken of two Bells, and have suggested that Bell must have been schizophrenic (Speiser, 1988).

I wish to argue that there was, unfortunately for us, but one Bell, and he was the sanest and most rational of men.

There is something else that I would like to do: I would like to convey a small sense of Bell's wonderful style, wit, and clarity. So to the extent possible I shall allow Bell to speak for himself. I shall read excerpts from Bell's articles on the foundations of quantum mechanics which pertain to our question. These articles are all collected in a marvelous book, *Speakable and Unspeakable in Quantum Mechanics* (Bell, 2004). I would urge all of you, and, indeed, anyone with an interest in physics, to read this book, and then read it again. I shall also have occasion to read from an interview Bell gave several years ago, to the philosopher Renee Weber.

1.3 The Impossibility of Hidden Variables

Bell's "On the impossible pilot wave" begins thus:

> When I was a student I had much difficulty with quantum mechanics. It was comforting to find that even Einstein had had such difficulties for a long time. Indeed they had led him to the heretical conclusion that something was missing in the theory: "I am, in fact, rather firmly convinced that the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this (theory) operates with an incomplete description of physical systems." (Bell, 1982; Bell, 2004, p. 159)

Einstein is expressing here the conviction that the supposedly novel quantum randomness will ultimately turn out to be of the same character as the familiar, normal, down-to-earth randomness exhibited, for example, in the behavior of a roulette wheel or a coin flip. The behavior appears random because there are too many relevant details to keep track of. If the quantum description could be completed by the incorporation of such details, the result would be called a hidden variable theory.

However, soon after the advent of quantum theory, any hidden variable account of quantum phenomena was mathematically "proven" to be impossible. Bell continues:

> Einstein did not seem to know that this possibility, of peaceful coexistence between quantum statistical predictions and a more complete theoretical description, had been disposed of with great rigour by J. von Neumann. I myself did not know von

Neumann's demonstration at first hand, for at that time it was available only in German, which I could not read. However I knew of it from the beautiful book by Born, *Natural Philosophy of Cause and Chance,* which was in fact one of the highlights of my physics education. Discussing how physics might develop Born wrote: "I expect...that we shall have to sacrifice some current ideas and to use still more abstract methods. However these are only opinions. A more concrete contribution to this question has been made by J. v. Neumann in his brilliant book, *Mathematische Grundlagen der Quantenmechanik.* ... The result is that ... no concealed parameters can be introduced with the help of which the indeterministic description could be transformed into a deterministic one. Hence if a future theory should be deterministic, it cannot be a modification of the present one but must be essentially different. How this could be possible without sacrificing a whole treasure of well established results I leave to the determinists to worry about."

Having read this, I relegated the question to the back of my mind and got on with more practical things. (Bell, 1982; Bell, 2004, p. 159)

1.4 The Impossible Accomplished

Bell continues :

But in 1952 I saw the impossible done. It was in papers by David Bohm. Bohm showed explicitly how parameters could indeed be introduced, into nonrelativistic wave mechanics, with the help of which the indeterministic description could be transformed into a deterministic one. More importantly, in my opinion, the subjectivity of the orthodox version, the necessary reference to the "observer," could be eliminated.

Moreover, the essential idea was one that had been advanced already by de Broglie in 1927, in his "pilot wave" picture. (Bell, 1982; Bell, 2004, p. 160)

Let me very briefly try to indicate the sort of thing Bell had in mind when objecting to the subjectivity of orthodox quantum theory, by means of a perhaps extreme example. Concerning the implications of quantum theory, in fact of Bell's theorem itself (about which more later), a very distinguished physicist once wrote that "the moon is demonstrably not there when nobody looks" (Mermin, 1981)[3].

[3] I can't resist including a mildly polemical rejoinder, from the late philosopher David Stove (1991, p. 99): "... If philosophy or religion prompts a person to deny or doubt that humans, or that kangaroos, are land-mammals, the only rational thing to do is to ignore him; and the same holds for science, too, whether past, present, or future.
I may be reminded that some respected physicists have said in recent years that something like Berkeleian idealism is actually a logical consequence of their best fundamental theories. (One of them wrote, for example: 'We now know that the moon is demonstrably not there when nobody looks'.) It would be irrational to believe this logical claim, but if it is true then it would be irrational to believe these physicists' best theories. Fundamental physical theories never say anything about a particular macroscopic physical object, such as the moon; but if they did say something about the moon, then they would say the same thing about all macroscopic physical objects, hence about all land-mammals, and hence about the particular land-mammal, Professor N. D. Mermin, who wrote the sentence I have just quoted. Now it

More Bell:

> Bohm's 1952 papers on quantum mechanics were for me a revelation. The elimination of indeterminism was very striking. But more important, it seemed to me, was the elimination of any need for a vague division of the world into "system" on the one hand, and "apparatus" or "observer" on the other. I have always felt since that people who have not grasped the ideas of those papers ... and unfortunately they remain the majority ... are handicapped in any discussion of the meaning of quantum mechanics. (Bell, 1984; Bell, 2004, p. 173)

Interview:

> In my opinion the picture which Bohm proposed then completely disposes of all the arguments that you will find among the great founding fathers of the subject—that in some way, quantum mechanics was a new departure of human thought which necessitated the introduction of the observer, which necessitated speculation about the role of consciousness and so on.
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> All those are simply refuted by Bohm's 1952 theory. In that theory you find a scheme of equations which completely reproduces all the experimental predictions of quantum mechanics and it simply does not need an observer. ... So I think that it is somewhat scandalous that this theory is so largely ignored in textbooks and is simply ignored by most physicists. They don't know about it.

1.5 How to do the Impossible

What does Bohm add to the standard quantum description? In a word, the particles themselves: For Bohm the so-called hidden variables are simply the positions of the particles of the quantum system, say the electrons of an atom. These particles move in a manner which is naturally choreographed by the wave function of the system. From the perspective of Bohm's theory, orthodox quantum mechanics leaves out the guts of the description, the very particles which combine to form everything we see around us.

Thus as applied to Bohm's theory, the terminology "hidden variables" seems rather inappropriate, suggesting as it does something exotic, artificial, and ad hoc. Bell:

> Absurdly, such theories are known as "hidden variable" theories. Absurdly, for there it is not in the wavefunction that one finds an image of the visible world, and the results of experiments, but in the complementary "hidden"(!) variables. Of course the extra variables are not confined to the visible "macroscopic" scale. For

may perhaps be true that Professor Mermin depends for his ease of mind on being an object of attention. This would not even be especially surprising, in view of the powerful emotional root which idealism has in common with religion. But that he depends for his very existence on being an object of attention, is entirely out of the question: it is much more likely (to say the least) that one or more of his scientific theories is wrong. Mammals are very complex, of course, and depend for their existence on a great many things; but somebody's looking at them is not among those things, and everybody knows this."

no sharp definition of such a scale could be made. The "microscopic" aspect of the complementary variables is indeed hidden from us. (Bell, 1989; Bell, 2004, p. 201)

Here Bell refers to the fact that in Bohm's theory the detailed trajectories of the microscopic particles are not observable. While this unobservability is a *consequence* of the very structure of Bohm's theory, many physicists quickly objected. After all, physics is about prediction, about observations, not about things which cannot be observed. Bell continues:

But to admit things not visible to the gross creatures that we are is, in my opinion, to show a decent humility, and not just a lamentable addiction to metaphysics. (Bell, 1989; Bell, 2004, p. 202)

1.6 The Accomplished Impossible is Ignored

The very existence of Bohm's theory, agreeing as it did in its predictions with those of orthodox quantum theory, quite naturally, under the circumstances, raised many questions for Bell:

But why then had Born not told me of this "pilot wave"? If only to point out what was wrong with it? Why did von Neumann not consider it? More extraordinarily, why did people go on producing "impossibility" proofs, after 1952, and as recently as 1978? When even Pauli, Rosenfeld, and Heisenberg, could produce no more devastating criticism of Bohm's version than to brand it as "metaphysical" and "ideological"? Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show us that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice? (Bell, 1982; Bell, 2004, p. 160)

1.7 What Went Wrong?

Of course, the most immediate question raised was, or should have been, What went wrong with the "proof"? Bell:

The realization that von Neumann's proof is of limited relevance has been gaining ground since the 1952 work of Bohm. However, it is far from universal. Moreover, the writer has not found in the literature any adequate analysis of what went wrong. Like all authors of noncommissioned reviews, he thinks that he can restate the position with such clarity and simplicity that all previous discussions will be eclipsed. (Bell, 1966; Bell, 2004, p. 2)

And Bell proceeded to do just that!

Bell analyzed von Neumann's proof as well as other proofs, found that they were based upon rather arbitrary assumptions or axioms, and focused on the the manner in which Bohm's theory violates these assumptions. In so doing he noticed that

> ...in this theory an explicit causal mechanism exists whereby the disposition of one piece of apparatus affects the results obtained with a distant piece.
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> Bohm of course was well aware of these features of his scheme, and has given them much attention. However, it must be stressed that, to the present writer's knowledge, there is no *proof* that *any* hidden variable account of quantum mechanics *must* have this extraordinary character. It would therefore be interesting, perhaps, to pursue some further "impossibility proofs," replacing the arbitrary axioms objected to above by some condition of locality, or of separability of distant systems. (Bell, 1966; Bell, 2004, p. 11)

1.8 Quantum Nonlocality

No sooner said than done! In fact, if we follow the publication dates, done before said—the EPR-Bell's theorem paper (Bell, 1964) in which it was done appeared almost two years before the paper (Bell, 1966) from which I was just quoting. Publication delay!

Bell interview:

> ...as a professional theoretical physicist I like the Bohm theory because it is sharp mathematics. I have there a model of the world in sharp mathematical terms that has this non-local feature. So when I first realized that, I asked: "Is that inevitable or could somebody smarter than Bohm have done it differently and avoided this non-locality?" That is the problem that the theorem is addressed to. The theorem says: "No! Even if you are smarter than Bohm, you will not get rid of non-locality," that any sharp mathematical formulation of what is going on will have that nonlocality.

Moreover, the nonlocality of Bohm's theory derives solely from the nonlocality built into the structure of standard quantum theory, as provided by a wave function on configuration space, an abstraction which, roughly speaking, combines—or binds—distant particles into a single irreducible reality. Bell:

> That the guiding wave, in the general case, propagates not in ordinary three-space but in a multidimensional-configuration space is the origin of the notorious 'nonlocality' of quantum mechanics. It is a merit of the de Broglie-Bohm version to bring this out so explicitly that it cannot be ignored. (Bell, 1980; Bell, 2004, p. 115)

Now the relevant experiments have been done (Aspect, Grangier and Roger, 1982), confirming the strange predictions to which Bell was led by his analysis of Bohm's theory. Where does this now leave us?

1.9 Lorentz Invariance

There is a basic problem: Bohm's theory violates Lorentz invariance, a central principle of physics. Nor can Bohm's theory be easily modified so that it becomes Lorentz invariant. The difficulty here arises from the fundamental tension, the *apparent* incompatibility, between nonlocality and Lorentz invariance. Bell interview:

> Now what is wrong with this theory, with David's theory? What is wrong with this theory is that it is not Lorentz-invariant. That's a very technical thing and most philosophers don't bother with Lorentz-invariance and in elementary quantum mechanics books the paradoxes that are presented have nothing to do with Lorentz-invariance.
>
> Those paradoxes are simply disposed of by the 1952 theory of Bohm, leaving as *the* [my emphasis] question, the question of Lorentz-invariance. So one of my missions in life is to get people to see that if they want to talk about the problems of quantum mechanics—the real problems of quantum mechanics—they must be talking about Lorentz-invariance.

And from the last sentence of (to my knowledge) Bell's last publication[4]—the LAST WORD, as it were:

Referring to Bohm's theory and to GRW theory (Ghirardi, Rimini and Weber, 1986; Bell, 1989)—a modification of quantum theory in which he became interested in his last years, Bell said

> The big question, in my opinion, is which, if either, of these two precise pictures can be redeveloped in a Lorentz invariant way. (Bell, 1990a; Bell, 2004, p. 230)

I believe that this really is the big question. And I urge it upon you. But I am afraid that in trying to answer this question, we shall miss Bell's help and inspiration very much indeed! And we shall miss Bell's marvelous style, his penetrating wit, and his brilliant clarity!

1.10 Epilogue

That's what I said some twenty years ago. Much the same could be said today. Bell's views on Bohm are more widely appreciated now. But they are not nearly as well appreciated as they should be. Bell was, after all, not an obscure writer, and he made his views perfectly clear. But he treated his readers seriously, and expected them to read him with the same care with which he wrote. In our age this sort of reading is all too rare.

As to the "big question," about Lorentz invariance, of which Bell spoke,

[4] At the time I wrote this my knowledge was in fact incomplete: It seems that Bell's last publication was La nouvelle cuisine (Bell, 1990b) and not Against measurement (Bell, 1990a).

significant progress has been made, of some of which, I'd like to think, Bell would have approved. Employing on a "flash" ontology, Roderich Tumulka has formulated a fully Lorentz invariant generalization of the GRW theory for entangled but noninteracting particles (Tumulka, 2006). Along rather different lines, by suitably employing past light cones, Bedingham et al. have provided a scheme for defining a Lorentz invariant redevelopment of GRW-type theories of a rather general character, including interacting quantum field theories (Bedingham et al., 2014). And for Bohmian mechanics there have been several proposals for Lorentz invariant generalizations, for example by exploiting the possibility of a covariant map from wave functions to space-like foliations of space-time (Dürr et al., 2013).

Tumulka's model is based on the ideas of Bell himself, and the peculiar flash ontology of the model was Bell's rather strange proposal for an ontology of the GRW theory (Bell, 1989; Bell, 2004, p. 205). Concerning this theory, Bell wrote that

> I am particularly struck by the fact that the model is as Lorentz invariant as it could be in the nonrelativistic version. It takes away the ground of my fear that any exact formulation of quantum mechanics must conflict with fundamental Lorentz invariance. (Bell, 1989; Bell, 2004, p. 209)

1.11 References

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