Are All Particles Real?

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Abstract

In Bohmian mechanics elementary particles exist objectively, as point particles moving according to a law determined by a wavefunction. In this context, questions as to whether the particles of a certain species are real—questions such as, Do photons exist? Electrons? Or just the quarks?—have a clear meaning. We explain that, whatever the answer, there is a corresponding Bohm-type theory, and no experiment can ever decide between these theories. Another question that has a clear meaning is whether particles are intrinsically distinguishable, i.e., whether particle world lines have labels indicating the species. We discuss the intriguing possibility that the answer is no, and particles are points—*just points*.

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1 Introduction

We address in this paper rather basic but intimidating questions about the ontology in Bohmian mechanics and similar theories, using two specific questions as a case study. What is intimidating about these questions is that they cannot be answered experimentally. However, as we shall explain, this does not mean they cannot be answered. Most, if not all, of what we point out in this paper has surely been known to some experts. However, we have found no clear discussion of the matter in the literature.

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Put succinctly, Bohmian mechanics is quantum mechanics with particle trajectories (e.g., Bohm, 1952a; Bohm, 1952b; Bell, 1966; Dürr, Goldstein & Zanghì, 1992; Dürr, Goldstein & Zanghì, 1996; Goldstein, 2001). It describes the motion of point particles in physical space $\mathbb{R}^3$. In the conventional form of Bohmian mechanics, the law of motion for the position $\boldsymbol{Q}_i(t)$ of the i-th particle of a system of N particles is

$$\frac{d\boldsymbol{Q}_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\psi_t^* \nabla_i \psi_t}{\psi_t^* \, \psi_t} \big(\boldsymbol{Q}_1(t), \ldots, \boldsymbol{Q}_N(t)\big) \tag{1}$$

where $\psi_t : \mathbb{R}^{3N} \to \mathbb{C}^\ell$ is a wavefunction obeying the Schrödinger equation

$$i\hbar \frac{\partial \psi_t}{\partial t} = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \Delta_i \psi_t + V \psi_t, \tag{2}$$

m_i is the mass of the i-th particle, numerator and denominator in (1) involve scalar products in the space $\mathbb{C}^\ell$ (corresponding to spin, quark color, quark flavor and similar degrees of freedom), Δ_i is the Laplacian with respect to $\boldsymbol{q}_i$ (the generic coordinates of the i-th particle), and finally V is the (possibly Hermitian $\ell \times \ell$-matrix-valued) potential function on $\mathbb{R}^{3N}$. The configuration $Q(t) = (\boldsymbol{Q}_1(t), \ldots, \boldsymbol{Q}_N(t))$ is random with distribution $|\psi_t|^2$ at every time t if $Q(0)$ is random with distribution $|\psi_0|^2$, as we shall assume in the following (Bohm, 1952a; Bohm, 1952b; Dürr, Goldstein & Zanghì, 1992). While the law of motion of Bohmian mechanics is highly non-Newtonian, Bohmian mechanics has in common with Newtonian mechanics that there are real particles—with actual positions—in contrast to most other versions of quantum mechanics.

One of the two questions we want to address in this paper is whether all elementary species (such as electron, muon, tauon, quark, photon, gluon, etc.) actually have particles. We have to explain what we mean by this. When we say that certain species have *no* particles, we have in mind the following modification of the theory defined by (1) and (2). Let $\mathscr{I} \subset \{1, \ldots, N\}$ be a nonempty index set (the set of the labels of all "real" particles), and stipulate that only $\#\mathscr{I}$ particles exist, labeled by the elements of $\mathscr{I}$ and moving according to

$$\frac{d\boldsymbol{Q}_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\langle \psi, \nabla_i \psi \rangle_{\mathscr{I}^c}}{\langle \psi, \psi \rangle_{\mathscr{I}^c}} \big(\boldsymbol{Q}_j(t) : j \in \mathscr{I}\big) \tag{3}$$

for $i \in \mathscr{I}$, where $\mathscr{I}^c = \{1, \ldots, N\} \setminus \mathscr{I}$, and

$$\langle \phi, \psi \rangle_{\mathscr{I}^c} (\boldsymbol{q}_j : j \in \mathscr{I}) := \int\limits_{\mathbb{R}^{3\#\mathscr{I}^c}} \Big(\prod_{k \in \mathscr{I}^c} d\boldsymbol{q}_k\Big) \, \phi^*(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N) \, \psi(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N) \tag{4}$$

is the partial scalar product between two wavefunctions ϕ and ψ, involving the inner product in $\mathbb{C}^\ell$ and integration over the coordinates labeled by $\mathscr{I}^c$ (of all "unreal particles"), yielding a complex function of the coordinates labeled by $\mathscr{I}$ (of the "real particles"). Equations (3) and (4) are completed by the unchanged Schrödinger equation (2).

This is a theory of $\#\mathscr{I}$ particles, even though (2) looks as if it is about N particles. We could take $\mathscr{I}$ to contain all electrons but not the quarks, or instead all quarks but not the electrons. As we will point out in Section 2, for both of these and many other choices, no conceivable experiment can detect a difference from conventional Bohmian mechanics as defined by (1) (or from one another), provided that the universe is in quantum equilibrium[1] (see Dürr, Goldstein & Zanghì, 1992) so that the configuration of the (real) particles has probability distribution

$$\rho = \langle \psi, \psi \rangle_{\mathscr{I}^c}. \tag{5}$$

We will also indicate in Section 2 why (1) may be more plausible than (3). Nevertheless, we believe that the impossibility, in quantum equilibrium, of conclusively rejecting (3) in favor of (1), or (1) in favor of (3), is but another instance of one of the *fundamental limitations of science*: that there may be distinct theories that are empirically indistinguishable.

The second question we consider in this paper is whether the configuration space should be, instead of $\mathbb{R}^{3N}$,

$${}^N\mathbb{R}^3 := \big\{ S \subseteq \mathbb{R}^3 : \#S = N \big\}, \tag{6}$$

the space of all N-element subsets of $\mathbb{R}^3$. ${}^N\mathbb{R}^3$ can also be identified with $\mathbb{R}^{3N}_{\neq}$ modulo permutations, where

$$\mathbb{R}^{3N}_{\neq} := \big\{ (\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N) \in \mathbb{R}^{3N} : \boldsymbol{q}_i \neq \boldsymbol{q}_j \; \forall i \neq j \big\} \tag{7}$$

is $\mathbb{R}^{3N}$ minus the coincidence configurations. The choice of ${}^N\mathbb{R}^3$ as configuration space corresponds to the notion that a configuration of N particles is a set of N points in physical space, with the points labeled in no way, neither by numbers $1, \ldots, N$, nor in the sense that there could be intrinsically different kinds of points in the world, such as electron points as distinct from muon points or quark points. (The configuration space ${}^N\mathbb{R}^3$ has nontrivial topology. See Dürr, Goldstein, Taylor, Tumulka & Zanghì (2005) for an investigation of the implications of this fact for Bohmian mechanics. This space has been suggested as the configuration space of N identical particles by Laidlaw & DeWitt (1971), Leinaas & Myrheim (1977), Nelson (1985), Brown, Sjöqvist & Bacciagaluppi (1999), and Dürr, Goldstein, Taylor, Tumulka & Zanghì (2005).)

To be specific, we have in mind again a concrete Bohm-type theory, combining the usual Schrödinger equation (2) with a modification of (1). Equation (1) itself does not define a dynamics for an unordered configuration, from ${}^N\mathbb{R}^3$, because different ways of

[1] In order to appreciate the strength of this indistinguishability, it is important to recognize that quantum equilibrium is absolute, not in the sense that the universe has to be in quantum equilibrium—it doesn't, although the correctness of the quantum mechanical predictions strongly suggests that it is—but in the sense that a universe in quantum equilibrium, like one in thermodynamic equilibrium, is pretty much stuck there and nothing we can do, nothing we can control, can get us out of it. Nonetheless, quantum non-equilibrium versions of these theories would be empirically distinguishable, from each other and from orthodox quantum theory; this sort of possibility has been explored by Valentini (2002).

numbering the N given points would (generically) lead to different trajectories of the particles. To obtain a well-defined dynamics on ${}^N\mathbb{R}^3$, we note that the right hand side of (1) can be written as

$$\frac{\boldsymbol{j}_i}{\rho}\big(\boldsymbol{Q}_1(t), \ldots, \boldsymbol{Q}_N(t)\big) \tag{8}$$

with

$$\boldsymbol{j}_i = \frac{\hbar}{m_i} \mathrm{Im}\, \psi^* \nabla_i \psi \tag{9}$$

and

$$\rho = \psi^* \, \psi. \tag{10}$$

Now symmetrizing $j = (\boldsymbol{j}_1, \ldots, \boldsymbol{j}_N)$ and ρ leads to a new equation of motion

$$\frac{d\boldsymbol{Q}_i}{dt} = \frac{\sum\limits_{\sigma \in S_N} \boldsymbol{j}_{\sigma(i)} \circ \sigma}{\sum\limits_{\sigma \in S_N} \rho \circ \sigma} \big(\boldsymbol{Q}_1(t), \ldots, \boldsymbol{Q}_N(t)\big) \tag{11}$$

where S_N is the set of permutations of $\{1, \ldots, N\}$ and

$$\sigma(\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_N) := (\boldsymbol{Q}_{\sigma^{-1}(1)}, \ldots, \boldsymbol{Q}_{\sigma^{-1}(N)})$$

so that $\boldsymbol{Q}_j$ is moved to the place $\sigma(j)$. Observe that renumbering the particles now does not change the trajectories.[2]

The difference between (1) and (11), which lead to different trajectories and thus define inequivalent theories, makes clear that in Bohmian mechanics the question whether particles are *just points*, represented by an element of ${}^N\mathbb{R}^3$, or *labeled points*, represented by an element of $\mathbb{R}^{3N}$, has a genuine physical meaning. Again, the question can never be decided experimentally, as we will explain in Section 3. There we will also say why this time the conventional formula (1) may be less plausible than its modification.

2 Do Photons Exist? Electrons? Quarks?

A relevant mathematical fact about Bohmian mechanics with a reduced number $\#\mathscr{I}$ of particles, as defined by (3), is the equivariance of the probability distribution (5): if

[2]Let us connect the transition from (1) to (11) with the "problem of recognition" raised by Brown, Elby, and Weingard (1996, p. 314) in a paper that is largely concerned with whether particle masses should be attributed to particle locations, or to the wave function, or to both—a question that does not concern us. Be that as it may, their argument concerning the problem of recognition, as we understand it, amounts to the observation that the labeled particles in the conventional Bohmian mechanics of distinguishable particles indeed *must be* labeled, in the sense of being intrinsically metaphysically distinct: otherwise which particle corresponds to which argument of the wave function and, consequently, to which velocity would be ambiguous. We note that it is exactly this ambiguity that renders (1) inadequate for unordered or unlabeled configurations, and that it is precisely this ambiguitity that is removed by the symmetrized dynamics (11): with (11) the positions of the particles alone suffice for deciding which one has which velocity.

the configuration $Q(t) = (\boldsymbol{Q}_j(t) : j \in \mathscr{I})$ is random with distribution $\rho_t = \langle \psi_t, \psi_t \rangle_{\mathscr{I}^c}$ at some time t, then this is so for all times t. This follows from the continuity equation for $\psi^* \psi$ implicit in the Schrödinger equation (2) by integrating over the coordinates $\boldsymbol{q}_k$ for $k \in \mathscr{I}^c$.

The distribution (5) is the basis of the empirical equivalence with conventional Bohmian mechanics as defined by (1): Suppose that the result of an experiment is recorded in the positions $\boldsymbol{Q}_j$ of the "real" particles, $j \in \mathscr{I}$. Since their distribution is the same as the marginal distribution of the $\boldsymbol{Q}_j$ in conventional Bohmian mechanics,

$$\langle \psi_t, \psi_t \rangle_{\mathscr{I}^c}(\boldsymbol{q}_j : j \in \mathscr{I}) = \int\limits_{\mathbb{R}^{3\#\mathscr{I}^c}} \Big(\prod_{k \in \mathscr{I}^c} d\boldsymbol{q}_k \Big) \big| \psi_t(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N) \big|^2, \tag{12}$$

there is no way to tell from the values of $\boldsymbol{Q}_j$, $j \in \mathscr{I}$, at one particular time whether they were generated using the reduced dynamics (3) or the conventional one (1). The same is true of any sequence of experiments, wherein the choice of the second experiment may even depend on the outcome of the first (Dürr, Goldstein & Zanghì, 1992). Thus, no conceivable experiment can decide between the two theories, provided that the outcomes of any conceivable experiment always get recorded in the $\boldsymbol{Q}_j$, $j \in \mathscr{I}$.[3]

An example of such a set $\mathscr{I}$ would be the set of all quarks. Pointer positions, ink marks on paper, and even the memory contents in a brain are reflected by the configuration of quarks. The same is true of electrons: if we know of an atom where its electrons are, we roughly know where its nucleus would have been if it existed. Even the positions of all photons alone would presumably suffice for fixing the macroscopic configuration, as every electron is believed to be surrounded by a cloud of photons, so that the photon configuration would roughly define the electron configuration (for quantum states relevant to ordinary macroscopic bodies) well enough to determine the values of macroscopic variables.[4]

[3]And provided that the $\boldsymbol{Q}_j$, $j \in \mathscr{I}$, always contain, in addition to the outcome, sufficient information to judge whether the experiment was properly conducted. All of this information would be available if the $\boldsymbol{Q}_j$, $j \in \mathscr{I}$, suffice to define the "macroscopic configuration."

Aside from this, there is a subtlety here that need not bother us for our considerations but should be mentioned nevertheless. It does not quite follow from equivariance that the configurations after the experiment will have the same distribution. This is illustrated by the following example that we owe to Tim Maudlin (private communication): consider Nelson's (1985) stochastic mechanics (more precisely, consider the formulation of stochastic mechanics due to Davidson (1979) for which the diffusion coefficient is a free parameter) with *extremely* large diffusion coefficient. Since the wave function of the universe must presumably be thought of as consisting of several packets that are very far apart in configuration space $\mathbb{R}^{3N}$, such as packets that correspond to unrealized outcomes of quantum measurements, the configuration $Q(t)$ will very probably also visit—in every second—those distant regions supporting the other packets, in some of which the dinosaurs have never become extinct. In this case the conditional distribution for the configuration after the experiment, conditional on that we did begin to perform the experiment, will be drastically different in stochastic mechanics from what it is in Bohmian mechanics. It is still the case that discrimination will not be possible—we probably will not even remember that we did that experiment in stochastic mechanics—but not exactly because of the simple reason that comes to mind at first.

[4]Here, too, there is a sublety into which we don't wish to delve in this paper. The point is that

We encounter here one of the fundamental limitations of science. There is no way of experimentally testing various possibilities for $\mathscr{I}$ against each other. There is no way of *conclusively establishing* what the true $\mathscr{I}$ is. This situation is probably new for most physicists;[5] most of us are used to contemplating rival theories that make similar predictions, but not rival theories that make exactly and perfectly the same predictions for all experiments. But the conclusion is simple: we have no alternative to accepting that we cannot finally know what $\mathscr{I}$ is, and we should simply admit this.

Nevertheless, this does not mean that all possibilities are equally plausible. On the contrary, many of the possibilities are ridiculous. For example, it is ridiculous to assume that everything presently outside the solar system is not real, though we (presently living humans) would not be able to find out (since outcomes of our experiments would be recorded in the particles inside the solar system). As another example, let $\mathscr{I}$ be a typical set with roughly $N/2$ elements; what makes this ridiculous is the complete arbitrariness of what is real and what is not. If we relax the assumption that $\mathscr{I}$ is a *fixed* set (how this can be done we describe in Appendix A), we find even more drastic examples: that everything outside the United States is not real (people in the US would not be able to find out), or that women are not real (men would not be able to find out), or—as a solipsistic kind of Bohmian theory—that only your brain, dear reader, is real (you would not be able to find out). Thus, some possibilities would be rejected by everyone, on basically the same grounds that one would reject solipsism: though it is logically possible and experimentally irrefutable, it is implausible—or whichever word you prefer.

It seems quite ridiculous as well, though perhaps not as obviously, to assume that quarks are not real. A Bohmian world in which quarks are not real would be an eccentric world, and a God who created such a world would certainly count as malicious. The only world that seems to us not eccentric at all is the one in which all particles are real. We mention some reasons: The Schrödinger equation (2) suggests that there are N particles. Why should there be coordinates in the wave function, varying in physical space, if there is no corresponding particle? On top of that, that all particles are real seems to be everybody's intuition, so much so that literally all physics texts talk about photons, electrons, and quarks, even those maintaining that microscopic reality does

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there is a sense in which, arguably, experimental discrimination is trivial in the only-photons version, because the predictions come out drastically wrong. It's not simply that pointers don't end up pointing with the right distribution; it could be argued that pointers don't end up pointing at all because there are no pointers—after all, pointers are not usually considered to be made of photons. Thus the only-photons theory would then seem to make no decent predictions at all. Of course a proponent of the only-photons theory wouldn't be inclined to accept that conclusion: he would, perhaps, appeal to the information encoded in the pattern of photon positions as the foundation of the predictions of his theory, using a map from photon configurations to (in fact, non-actual) electron configurations to results of experiments. For such a person, regardless of whether he is right or wrong in behaving as he does, experimental discrimination between his theory and a more normal theory would be impossible.

[5]The situation is in fact not new to researchers in Bohmian mechanics, as Bohmian mechanics is known to be empirically equivalent to other dynamics on $\mathbb{R}^{3N}$ that make the $|\psi|^2$ distribution equivariant, such as Nelson's (1985) stochastic mechanics and the velocity laws studied by Deotto and Ghirardi (1998).

not exist. Another thing to keep in mind is that you may have found it difficult back in Section 1 to understand the strange thought that some particles are "real" and some are "not real"; in any case, we found it difficult to explain and even to merely express. This may indicate that (3) is not a very natural idea.

We finally remark that one could perhaps imagine that there may be compelling reasons (of a mathematical or physical kind) precluding certain species from having particles. These would of course be reasons for preferring (3) to (1); presently, however, we do not know of any such reasons.

3 Are Particles Just Points?

We now turn to a discussion of the symmetrized law of motion (11). The empirical equivalence between symmetrized Bohmian mechanics, taking place in ${}^N\mathbb{R}^3$, and conventional Bohmian mechanics as defined by (1) is again based on equivariance. As is easily checked, (11) implies equivariance of the distribution

$$\rho_{\mathrm{sym}} = \sum_{\sigma \in S_N} \rho \circ \sigma \,, \tag{13}$$

a symmetric distribution on $\mathbb{R}^{3N}$ which thus defines a distribution on ${}^N\mathbb{R}^3$, which again equals a marginal of the distribution (10) of the conventional configuration $(\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_N) \in \mathbb{R}^{3N}$, namely the distribution of the set $\{\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_N\}$ disregarding the labels. Now empirical equivalence follows, by a reasoning similar to the one following (12), from the fact that outcomes of all conceivable experiments will be recorded in the unordered configuration $\{\boldsymbol{Q}_1, \ldots, \boldsymbol{Q}_N\}$. To illustrate this fact, we may imagine the outcome as given by the orientation of a pointer on a scale; as the pointer consists of a huge number of electrons and quarks, for reading off the orientation of the pointer we need not be explicitly told which points are the electrons and which are the quarks. Moreover, it is important to bear in mind that our assessment of which particles are quarks and which are electrons presumably could not be based on any sort of direct access to the particle's intrinsic nature, but rather must be based on information about the particle's behavior, reflected in the overall configuration of the particles.

We have two possibilities: particles belonging to different species may be metaphysically different, i.e., electron points may be different from quark points or photon points, or, alternatively, they may all be just points. As in the situation of the previous section, the impossibility of deciding experimentally between these possibilities is a fundamental limitation of science. A choice can only be based on theoretical considerations, and this time both possibilities seem plausible enough to be acceptable.

We would like to point out, though, that the possibility that all particles are just points, first implicitly suggested in Bell's (1986) seminal paper, is quite attractive, more attractive than it may appear at first. And not merely because of metaphysical simplicity. As we have already indicated, if, say, electrons and muons were different kinds of points then this difference in the nature of these points would be in no way directly accessible to us. Our decision as to whether a given particle is an electron or a muon would be based on its behavior under the conditions (such as external fields) we impose, i.e., based on its trajectory.

We add another thought. Sometimes progress in theoretical physics forces us to regard what was previously considered two species as two quantum states of the same species. For instance, proton and neutron appeared as two species, but in fact are two states of a three-quark system. The most radical development possible in this direction, considered by Goldstein, Taylor, Tumulka & Zanghì (2004), would be that all of what we presently consider different species are just different states of the same species—which would of course force us to adopt the symmetrized dynamics (11), and would suggest against (3).

4 Conclusions

We have made the observation that the following two questions have a clear physical meaning in Bohmian mechanics, however obscure they may be in other versions of quantum mechanics: Do all particle species have particles? And, do the particles have labeled or unlabeled world lines? These questions are about various conceivable equations of motion that we have explicitly specified. We have underlined that the various theories are empirically equivalent (provided the set of real particles is not too small), so that any answer to these questions would have to be grounded in purely theoretical, possibly philosophical, considerations. We have argued that the most convincing answer to the first question is that all species have particles, unless this is precluded by compelling mathematical or physical reasons yet to be discovered. Concerning the second question, we have suggested that the most attractive possibility is that world lines are unlabeled: what are normally regarded as distinguishable particles are better regarded as intrinsically indistinguishable.

Bell (1986) expressed concern about the empirical equivalence of various choices of what is real, in the context of his lattice model analogous to Bohmian mechanics. The situation may be much better, however, than Bell thought: concerning our question as to which particles are real, the possibilities differ greatly in plausibility, and one of them seems clearly more natural than all others—so there is not so much to be concerned about.

Appendix A

Here is a brief outline of how the dynamics (3) can be modified in order to deal with a variable set $\mathscr{I}$. One example to have in mind is that, so to speak, $\mathscr{I}(t)$ contains all particles in a particular region $R \subseteq \mathbb{R}^3$ of physical space. More generally, we can let $\mathscr{I}$ depend on time and even on the configuration.

To this end, we start with a partition $\big\{S_{\mathscr{I}} : \mathscr{I} \subseteq \{1, \ldots, N\}\big\}$ of configuration space-time $\mathbb{R}^{3N} \times \mathbb{R}$ indexed by all sets of particle labels. Equivalently, we can start with a configuration-dependent set $\mathscr{I}(q,t) \subseteq \{1, \ldots, N\}$ indicating which particles are real, specified by a function from configuration space-time $\mathbb{R}^{3N} \times \mathbb{R}$ to the power set of $\{1, \ldots, N\}$; the partition is then formed by the level sets of this function. In the example case in which only the particles in the region R are real, we would set $S_{\mathscr{I}} = \{(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N, t) : \boldsymbol{q}_j \in R, \boldsymbol{q}_k \notin R \text{ for } j \in \mathscr{I}, k \in \mathscr{I}^c\}$. Since the number of real particles can change, the dynamics of the configuration takes place in the disjoint union[6]

$$\mathcal{Q} = \bigcup_{\mathscr{I} \subseteq \{1,\ldots,N\}} (\mathbb{R}^3)^{\mathscr{I}}.$$

Let $\pi_{\mathscr{I}} : S_{\mathscr{I}} \to (\mathbb{R}^3)^{\mathscr{I}} \times \mathbb{R}$ be the projection $\pi_{\mathscr{I}}(\boldsymbol{q}_1, \ldots, \boldsymbol{q}_N, t) = (\boldsymbol{q}_j : j \in \mathscr{I}, t)$, and $\pi : \mathbb{R}^{3N} \times \mathbb{R} \to \mathcal{Q} \times \mathbb{R}$ the combination of the $\pi_{\mathscr{I}}$, i.e., $\pi(q,t) = \pi_{\mathscr{I}}(q,t)$ when $(q,t) \in S_{\mathscr{I}}$.

The dynamics for $Q(t)$ in $\mathcal{Q}$ we now define is the Markovization of the stochastic process $\pi(\tilde{Q}(t), t)$, where $\tilde{Q}(t)$ denotes the conventional Bohmian motion in $\mathbb{R}^{3N}$ defined by (1). $Q(t)$ will consist of deterministic trajectories interrupted by stochastic jumps that change the number (or labels) of the real particles. See Dürr, Goldstein, Tumulka & Zanghì (2004) for the general theory of particle creation in Bohmian mechanics by means of stochastic jumps. The deterministic motion is defined by

$$\frac{d\boldsymbol{Q}_i}{dt} = \frac{\boldsymbol{j}_i(Q(t),t)}{\rho(Q(t),t)}, \tag{14}$$

where, for $q \in (\mathbb{R}^3)^{\mathscr{I}}$,

$$\boldsymbol{j}_i(q,t) = \int\limits_{\pi_{\mathscr{I}}^{-1}(q,t)} \Big(\prod_{k \in \mathscr{I}^c} d\boldsymbol{q}_k\Big) \tfrac{\hbar}{m_i} \operatorname{Im} \psi_t^* \nabla_i \psi_t \tag{15}$$

is the probability current and

$$\rho(q,t) = \int\limits_{\pi_{\mathscr{I}}^{-1}(q,t)} \Big(\prod_{k \in \mathscr{I}^c} d\boldsymbol{q}_k\Big) \psi_t^* \psi_t \tag{16}$$

is the probability density on $\mathcal{Q}$. The rate of jumping from $Q \in \mathcal{Q}$ to a configuration $q' \in \mathcal{Q}$ (in which the particles from $\mathscr{I}'$ are real) is

$$\sigma_t(dq', Q) = \frac{J^+(dq', dQ, dt)}{\rho(Q,t)\, dQ\, dt}, \tag{17}$$

where $x^+ = \max(x, 0)$, and $J(dq', dQ, dt)$ is the probability flow, in $\mathbb{R}^{3N} \times \mathbb{R}$, in the time element dt from $S_{\mathscr{I}}$ to $S_{\mathscr{I}'}$ across the boundary $\partial S_{\mathscr{I}}$ at volume elements dq' and dQ,

$$J(dq', dQ, dt) = \int\limits_{D(dq', dQ, dt)} dA \Big(n_0 \psi_t^* \psi_t + \sum_{i=1}^{N} \tfrac{\hbar}{m_i} \operatorname{Im} \psi_t^* \boldsymbol{n}_i \cdot \nabla_i \psi_t\Big) \tag{18}$$

[6] In what follows, $(\mathbb{R}^3)^{\mathscr{I}} = \{(\boldsymbol{q}_j : j \in \mathscr{I})\}$, with each $\boldsymbol{q}_j \in \mathbb{R}^3$, is the set of functions from $\mathscr{I}$ to $\mathbb{R}^3$. $(\mathbb{R}^3)^{\mathscr{I}}$ is isomorphic to $\mathbb{R}^{3\#\mathscr{I}}$, but $(\mathbb{R}^3)^{\mathscr{I}} \neq (\mathbb{R}^3)^{\mathscr{I}'}$ even for $\#\mathscr{I} = \#\mathscr{I}'$ (unless of course $\mathscr{I} = \mathscr{I}'$).

with integration domain $D(dq', dQ, dt) = \pi_{\mathscr{I}}^{-1}(dQ \times dt) \cap \pi_{\mathscr{I}'}^{-1}(dq' \times dt) \cap \partial S_{\mathscr{I}} \cap \partial S_{\mathscr{I}'}$, dA the surface element on the boundary and $(n_0, \boldsymbol{n}_1, \ldots, \boldsymbol{n}_N)$ the unit normal vector on the boundary pointing from $S_{\mathscr{I}}$ to $S_{\mathscr{I}'}$.

By standard arguments (Dürr, Goldstein, Tumulka & Zanghì, 2004), one shows that the distribution (16) is equivariant.

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