ON A REALISTIC THEORY FOR QUANTUM PHYSICS

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§1. Introduction

We discuss some ideas about quantum physics which we think are of relevance for the future evolution of the field. These ideas, though old, are either unknown or misunderstood. Our point here is that a strong realistic position has consequences: it offers a completely natural understanding of "standard quantum mechanics"; it fully reveals the nonlocal character of nature and it guides the search for a fundamental unified theory of the microscopic and macroscopic world.

We wish to dedicate this paper to D. Bohm, J.S. Bell and to the memory of A. Einstein.

§2. Hidden variables

Today a realistic theory for quantum physics is called a hidden variables theory. Hidden variables are variables which are not contained in standard quantum mechanics, but are nevertheless regarded as being responsible for the outcomes of measurements and their statistical character. Admitting hidden variables means admitting that quantum mechanics is not complete. To prove the necessity of hidden variables Einstein, Schroedinger and others constructed Gedanken-experiments showing that quantum mechanics does not satisfy a basic

requirement of completeness: A physical theory must contain a description of all physical systems with which it means to deal, including the systems involved in the measurement process, in such a way that no contradiction arises with its basic principles. That the quantum mechanical treatment of the measurement process (see e.g. [1] or [2]) does lead to contradictions has been emphasized by Bell [3] and Wigner [4].

§3. Beables

Bell, arguing against the observable-based quantum theory introduced the name of "be-able", as opposed to "observ-able", to describe what is physically real and whose existence does not depend on observation. The observables, which describe the outcomes of measurements, should be a construct of beables. The introduction of beables, as building blocks of any reasonable physical theory, makes explicit the "classical terms" which, according to Bohr, are to express the results of measurements, and eliminates the need for the artificial distinction between "observer" and "observed", between ordinary physical processes and measurements [5].

If one accepts the need for a realistic theory, one is confronted with "impossibility proofs" which, like von Neumann's or Gleason's, attempt to show that all hidden variables theories strongly violate quantum mechanical predictions [6]. It is therefore worthwhile to focus on Bohm's quantum theory which is a realistic reformulation of nonrelativistic quantum physics [7,8].

§4. Bohm's theory

Bohm's theory is an elaboration of de Broglie's pilot wave theory [9]: The state of an N particle system is given by

$$(\vec{q}_1, \ldots, \vec{q}_N, \psi(\vec{q}_1, \ldots, \vec{q}_N)) \in \mathbb{R}^{3N} \times L^2(\mathbb{R}^{3N}), \tag{1}$$

the positions of all the particles and the wave function of the system, with time evolution

$$i\hbar \frac{d\psi}{dt} = H\psi \tag{2}$$

$$\frac{d\vec{q}_n}{dt} = \vec{V}_n(\vec{q}_1, \ldots, \vec{q}_N; t), \quad n = 1, \ldots, N. \tag{3}$$

Here H is the hamiltonian of the system and in simple cases has the form

$$H = \sum_{n=1}^{N} -\frac{\hbar^2}{2m_n}\Delta_n + \sum_{n<j} U(|\vec{q}_n - \vec{q}_j|).$$

The vector fields $\vec{V}_1, \ldots, \vec{V}_N$ are obtained by comparing the continuity equation for the probability density associated with (3),

$$\frac{\partial \rho}{\partial t} + \sum_{n=1}^{N} \vec{\nabla}_n \cdot (\vec{V}_n \rho) = 0, \tag{4}$$

with the continuity equation for $P \equiv |\psi|^2$ arising from (2)

$$\frac{\partial P}{\partial t} + \sum_{n=1}^{N} \vec{\nabla}_n \cdot \vec{\Im}_n = 0, \tag{5}$$

$\vec{\Im}_n$ being given by

$$\vec{\Im}_n = \frac{\hbar}{2im_n}[\psi^* \vec{\nabla}_n \psi - (\vec{\nabla}_n \psi^*)\psi].$$

Thus we define $\vec{V}_n$:

$$\vec{V}_n = \frac{1}{P}\vec{\Im}_n. \tag{6}$$

Note that with $\psi = \sqrt{P}\, e^{\frac{i}{\hbar}S}$, P and S being real, this becomes

$$\vec{V}_n = \frac{1}{m_n}\vec{\nabla}_n S.$$

Thus the motion of $\vec{q}_n(t)$, the position of the n-th particle, is piloted by $\psi(t)$, which evolves autonomously according to Schroedinger's equation. Moreover if $\rho(\vec{q}_1, \ldots, \vec{q}_N; t)$, the probability density distribution of $\vec{q}_1(t), \ldots, \vec{q}_N(t)$, is equal to $|\psi(\vec{q}_1, \ldots, \vec{q}_N; t)|^2$ at some time t then it follows from (4) and (6) that this remains so provided no new information has been gathered. If $\rho = |\psi|^2$ Bohm's theory agrees with the quantum mechanical predictions concerning the outcomes of position measurements and thus with all quantum mechanical predictions: The relevant macroscopic variables describing pointer positions, computer printouts, etc. are functions of the microscopic coordinates $\vec{q}_1, \ldots, \vec{q}_N$ (among which are those giving the microscopic configuration of the apparatus). To every measurement of a quantum mechanical observable there corresponds in Bohm's theory a random variable in configuration space (and there is, in fact, no conflict with von Neumann's or Gleason's impossibility proofs).

§5. Quantum equilibrium

One has to reflect upon the problem that $\rho(\vec{q}_1, \ldots, \vec{q}_N; t) \approx |\psi(\vec{q}_1, \ldots, \vec{q}_N; t)|^2$ in all physically relevant situations, so that the quantum mechanical predictions are not grossly violated. The dynamical system (2), (3) is deterministic: for given initial data $(\vec{q}_1(0), \ldots, \vec{q}_N(0); \psi(0))$ the future evolution is completely specified. Probabilities enter only through ignorance of the initial conditions, therefore $\rho \approx |\psi|^2$ cannot be assumed as a constraint.

The situation here is similar to that of classical statistical mechanics which makes arbitrary use of the Gibbs distribution more or less taking for granted the validity of the ergodic hypothesis. The hypothesis that $\rho \approx |\psi|^2$ in all relevant physical situations, which we call the hypothesis of "quantum equilibrium", obviously needs some justification. We refer to our forthcoming paper [10] for a careful discussion of this point, limiting ourselves here to some heuristic considerations.

The validity of the Gibbs distribution in classical statistical mechanics was understood by Boltzmann in terms of the chaotic behaviour of the phase-trajectory of a large system, which in the course of time should fill the surface of constant energy. The modern way of coming to grips with this idea is through the mathematical notions of ergodicity, mixing, exponential instability, etc., [11]. It is not unreasonable to expect that proper generalizations of these notions to time-dependent situations will show that in the appropriate sense $|\psi|^2$ is the ergodic measure of the dynamical system $\vec{q}_1(t), \ldots, \vec{q}_N(t)$. An indication of this is the fact that as soon as ψ is not an eigenstate of the hamiltonian, the dynamical system (3) becomes very sensitive to changes in the initial conditions.

§6. Stochastic models

As in nonequilibrium statistical mechanics the general validity of quantum equilibrium can presumably be also understood in terms of random disturbances acting on the system and driving the initial distribution ρ to the equilibrium value $|\psi|^2$. A phenomenological way of representing this effect is to consider instead of (3) a well mixing "stochastic flow" whose equilibrium distribution is $|\psi|^2$. A simple choice is to consider the class of diffusions parametrized by the diffusion constants ν_n,

$$d\vec{q}_n = \vec{V}_n^{(\nu)}\, dt + \nu_n\, d\, W_n \tag{7}$$

with W_n independent Wiener processes and ν_n unknown parameters reflecting the effect of the environment. (One might also regard ν_n as representing the strength of some "intrinsic" randomness [12,13].) Consistency with (5) requires that the probability current of (7) (a function of the probability density ρ)

$$\vec{\Im}_n^{(\nu)}(\rho) = \vec{V}_n^{(\nu)} \rho - \frac{1}{2} \nu_n^2 \vec{\nabla}_n \rho, \tag{8}$$

be equal in equilibrium, $\rho = P \equiv |\psi|^2$, to the quantum current appearing in (5)

$$\vec{\Im}_n^{(\nu)}(P) = \vec{\Im}_n ,$$

from which we obtain that

$$\vec{V}_n^{(\nu)} = \frac{1}{P}\left(\vec{\Im}_n + \frac{1}{2} \nu_n^2 \vec{\nabla}_n P\right). \tag{9}$$

It is remarkable that for all $\nu_n > 0$ there is a natural stochastic version of acceleration $\vec{a}_n^{(\nu)}$ (for nondifferentiable trajectories) such that at equilibrium a stochastic version of Newton's law holds:

$$m_n \vec{a}_n^{(\nu)} = - \vec{\nabla}_n \sum_{j \neq n} V(|\vec{q}_n - \vec{q}_j|). \tag{10}$$

Nelson [14] based a reformulation of quantum mechanics on (10) ("Stochastic mechanics", see also [15,16]).

§7. Measurement

In order to understand the predictions of Bohm's theory - and its stochastic generalizations - one must understand the interplay of coherence and dissipation (destruction of coherence due to interaction with the envirroment) and the role of the position as beable. One may see for example how the usual axioms of quantum measurement (such as:

The results of a measurement of an observable are represented by the eigenvalues of the asssociated self-adjoint operator ... .
A measurement always causes the system to jump into an eigenstate of the observable that is being measured...)

arise as an elementary consequence [8,13,17,18]. This shows that the "measurement problem" can be solved in a very simple way: Bohm's theory fulfils the requirement of completeness (§1).

Let us mention here the case of a position measurement: It is well known that a sharp position measurement cannot be performed without a large change in the wave function. This is more or less clear in standard quantum mechanics, and in any case taken for granted there - it is embodied in the Heisenberg uncertainty principle. It is even clearer in Bohm's theory where it also explains as well why trajectories are not measurable: Any attempt to measure the position of a particle at a certain time affects the wave function thereby influencing the future dynamics of the particle.

§8. Nonlocality

Another even more striking feature of Bohm's theory is the following: Since the wave function lives on configuration space, making an observation on one particle affects the wave function everywhere and thus by (6) and (3) the trajectories of all others particles. This "nonlocality" is clearly implicit in quantum theory and led Einstein, who believed that nature is local, to the E.P.R. gedanken-experiment [19] and to the conclusion that quantum theory is incomplete. Einstein implicitly assumed that it is the statistical character of quantum mechanics - for Einstein a reflection of incompleteness - which is responsible for this nonlocality. Bell's analysis [20] of Bohm's spin version of the EPR experiment shows that this implication is wrong, in particular that all realistic theories must be nonlocal if they are to agree with quantum mechanical predictions. We may summarize the situation as follows: (A) EPR show that locality and quantum mechanics imply hidden variables; (B) Bell shows that locality and quantum mechanics imply there are no hidden variables. One then concludes from (A) and (B) that quantum mechanics or any other theory whose predictions agree with those of quantum mechanics, is nonlocal. Moreover, since experiments [21] verify the quantum mechanical predictions, nature is nonlocal. (It is amusing to remark that the E.P.R. argument already provoked Schroedinger to question whether quantum theory conforms with nature [22].)

But how can this nonlocality be compatible with the fact that information cannot be transmitted faster than light? Though the realistic theories discussed so far are nonrelativistic, we can get a handle on this question by looking carefully at the nature of the nonlocality occurring in them. The impossibility of observing the trajectory of one particle without disturbing it (see above) precludes an action at distance from having observable effects.

The problem of the compatibility of nonlocality and Lorentz invariance is more subtle.

It is not a priori clear how to formulate a fundamental relativistic theory embodying nonlocality. Standard quantum mechanics overcomes this problem through a formulation based solely on "local observables" which are functions of some local relativistic quantum field (see below) and no conflict between relativity and nonlocality seems to arise. However the problem might merely be hidden and may be partly reflected in the mathematical difficulties encountered in formulating a satisfactory and nontrivial relativistic field theory (in particular when general covariance is required). On the level of a "Beable theory" the situation is unclear.

§9. Relativistic beables

We wish to address the problem of the formulation of a realistic relativistic quantum theory. We begin by discussing generalizations of Bohm's theory.

The variables which represent physical reality in Bohm's quantum theory are the positions $\vec{q}_1(t), \ldots, \vec{q}_N(t)$ of the particles and the wave function $\Psi(\vec{q}_1, \ldots, \vec{q}_N; t)$. Thus the "Schroedinger representation" attains special significance among all other mathematically equivalent representations of the wave function. It is thus natural to generalize Bohm's theory to an arbitrary representation:

Given a Hilbert space of states H and a unitary evolution U_t, define the "configuration space" Q, the space of the possible values of beables a, b, c, ..., as the space of all simultaneous eigenvalues of a complete commuting set of self-adjoint operators A, B, C, ... which in some way connect naturally to measurement, in the sense that the eigenvalues may really be values of beables. Then a Bohm-type theory can be obtained with a wave function $\Psi(a, b, c, \ldots) \in L^2(Q)$ guiding the motion of a(t), b(t), c(t), ... provided that all the observables – describing outcomes of measurements and represented by self-adjoint operators not commuting with A, B, C, ... – can be expressed in a physically sensible way as functions of the microscopic variables a, b, c, ... $\in$ Q. (We here ignore the possibility, which in fact arises in connection with the treatment of spin, that only a subset of a complete commuting set may correspond to beables.) Clearly the position operator is the only natural choice in nonrelativistic quantum physics. Unfortunately it seems that there is no natural choice in the relativistic case: Given a relativistic quantum field theory one must somehow choose a commuting set of self-adjoint operators whose eigenvalues are to have the status of beables. Here one is being guided more by mathematical analogies than by physical principles - maybe the "natural beables" are not eigenvalues of self-adjoint operators.

(Quasi-) Bohm-type theories have been constructed by Guerra and Ruggiero [23], Nelson

[24] (in the stochastic case: $\nu > 0$) and by Bohm [17,25] (in the deterministic case: $\nu = 0$) for the scalar bosonic free field, and by Bell [26] for Fermi fields (quasi because our proviso about "outcomes of measurements" has not been checked). In the first case the operator was chosen to be the field Φ itself (the natural mathematical generalization of position) whereas Bell used the fermionic number operator N. Bell's choice seems somehow more physical since the distribution of fermions in space relates directly to the macroscopic world: it determines the position of instruments (and the ink one puts down on papers to record, which, nobody doubts, should be real).

For our purposes we consider more detailed the bosonic case in Bohm's formulation.

§10. Bohm's quantum field theory

We now present Bohm's formulation for bosonic fields. Consider the quantum scalar field Φ. Since Φ commutes on spacelike surfaces $\sigma \subset \mathbb{R}^4$,

$$[\Phi(x), \Phi(x')] = 0 \qquad x, x' \in \sigma \subset \mathbb{R}^4, \tag{11}$$

it is convinient to fix a frame, i.e. a foliation of space-time into parallel spacelike hyperplanes, copies of $\mathbb{R}^3$, and to take as configuration space Q, the classical field space of generalized field configurations on these hyperplanes. The dynamical law (the infinitesimal form of the representation in $L^2(Q)$ of the time translation subgroup of the Poincaré group) reads

$$i\,\frac{d\psi_t(f)}{dt} = H\,\psi_t(f), \quad f \in Q \quad (\hbar{\equiv}1), \tag{12}$$

where H is an infinite dimensional differential operator; in the massless free case for example it is given by the formal operator in $L^2(Q)$

$$H = \tfrac{1}{2}\int d^3x\,\Big[-\frac{\delta^2}{\delta f(\vec{x})^2} + (\vec{\nabla} f(\vec{x}))^2\Big]. \tag{13}$$

Writing

$$\psi_t(f) = \sqrt{P_t(f)}\, e^{iS_t(f)}$$

and proceeding as in the nonrelativistic case one obtains the dynamical law guiding the evolution of the field's configuration f,

$$\frac{df_t(\vec{x})}{dt} = \frac{\delta}{\delta f_t(\vec{x})} S_t(f_t). \tag{14}$$

The field configurations $f_t(\vec{x})$ are the local beables of the theory; ψ_t, though nonlocal, is also regarded as a beable. The dynamical system (12), (14) should be thought of as a natural mathematical generalization of (2), (3).

When all the relevant fields (describing matter and radiation) are taken into account in such a way that, at least in principle, a purely field-theoretic description of any physical situation of interest (e.g. a measurement) exists, then all the local quantum observables can be expressed in terms of the local beables. Then, as in the nonrelativistic case, it will be possible to explain all the quantum rules and to recover all quantum mechanical predictions by taking into account the interplay of coherence and dissipation [25].

The theory also enhances our understanding of the so called "wave-particle duality". The quanta appear in fact as "continuous" structures in the field which nevertheless display a discrete character in processes involving the interaction of the (radiation) field with matter [25].

A stochastic version of this theory may be developed by considering instead of (14) a suitable infinite dimensional diffusion process and proceeding as in §6.

§11. The problem of relativistic invariance

Unfortunately the theory presented in §10 violates the relativity principle. This may be seen, for example, in the evolution of the field f_t guided by the vacuum wave function Ω. Since for the vacuum wave function S(f) is zero, i.e. $\Omega(f) = \sqrt{P(f)}$, the dynamical law for f_t is trivial,

$$\frac{df_t(\vec{x})}{dt} = 0. \tag{15}$$

However a Lorentz transformation of the time evolution (15) would be nontrivial, though the vacuum wave function would be unaffected. One concludes that there is a privileged reference frame for this theory. Note that the situation is curious since the predictions of the theory are in agreement with the quantum ones (the Lorentz invariant character of which is manifest in the Heisenberg picture, since the quantum field operators satisfy familiar, or at least manifestly, covariant equations). Thus nonlocality is clearly compatible with observational Lorentz invariance.

It might be useful at this point to underline the common origin of nonlocality and violation of Lorentz invariance in this theory: The foliation of Minkowski space-time into parallel space-like hyperplanes is an additional geometrical structure providing an absolute standard of simultaneity for the nonlocal connections between separated regions of space-time. A local measurement of the field f affects the wave-function and thus the field f globally on a whole simultaneity layer.

It might be that the breaking of Lorentz invariance at the fundamental level is forced by the structure of relativistic quantum theory, in particular by the way in which the wave function embodies the nonlocal character of nature. Such a conclusion is also suggested when one considers the fermionic number operator as a beable [26]. (For the stochastic version of the bosonic case one might look at the averaged field $<f_t(\vec{x})>$ for which eq. (15) holds and argue as above.) But such a conclusion may be too hasty for it might still be possible to introduce beables into quantum theory in a manner compatible with the relativity principle. (It is, of course, not necessary that every quantum theory have a satisfactory realistic reformulation, only those to be taken seriously as fundamental theories.)

A suggestion along these lines arises from the following observation: Consider the quantum field operators Φ_x, $\Phi_{x'}$, Φ_y and $\Phi_{y'}$ localized around the space-time points x, x′, y and y′ chosen in such a way that x and x′ are space-like separeted from y and y′ and x′ (y′) is in the absolute future of x (y). Then in general

$$[\Phi_x, \Phi_y] = [\Phi_x, \Phi_{y'}] = [\phi_{x'}, \phi_y] = [\phi_{x'}, \phi_{y'}] = 0\,,$$
$$[\phi_x, \phi_{x'}] \neq 0 \qquad\qquad [\phi_y, \phi_{y'}] \neq 0\,. \tag{16}$$

If one now associates beables f_x, $f_{x'}$, f_y and $f_{y'}$ with the operators Φ_x, $\Phi_{x'}$, Φ_y and $\Phi_{y'}$, one expects in general that the joint probability distributions of pairs of f's corresponding to commuting Φ's cannot all agree with the corresponding quantum mechanical distributions. The commutativity structure (16) is critical for the proof of so called "no-go theorems" for hidden variables. This suggests that a given foliation of space-time into simultaneity layers may determine which families of beables are jointly directly measurable, in the sense that a joint measurement of the corresponding Φ's reveals the joint values of these beables; in particular, the joint distribution of jointly directly measurable beables must agree with the quantum mechanical distributions [27]. (The reader who detects a hint of contradiction should pay careful attention to nonlocality.) Note that this agreement between the beable and the quantum mechanical joint probability distributions on each simultaneity layer should allow one to recover the quantum mechanical predictions for the outcomes of all (joint) measurements, since the results of measurements in different space-time regions can all be recorded in a single future region.

§12. Nonlocality and Lorentz invariance: a possible route

It seems that the only way to reconcile the relativity principle with the existence of special simultaneity layers is to give them a dynamical origin: The foliation of space-time into simultaneity layers, as an additional element of geometrical structure, should be determined, like the metric in general relativity, by what in quantum mechanics represents the "distribution of matter": the field f and the wave function ψ.

In order to proceed one has first to adapt the Schroedinger equation (12) to an arbitrary foliation $\{\sigma\}$ of space-time into nonintersecting hypersurfaces σ's (which need not be hyperplanes). One obtains in this way a Tomonaga form [28] for the Schroedinger equation:

$$i\,\frac{\delta}{\delta\sigma}\,\psi(\sigma,f_\sigma) = H_{\{\sigma\}}\psi(\sigma, f_\sigma), \tag{17}$$

where $\frac{\delta}{\delta\sigma}$ is a functional derivative in the space of surfaces which , for a fixed foliation $\{\sigma\}$ becomes the directional derivative arising from the integral curves of the vector field $N_{\{\sigma\}} = n^{\mu}_{\{\sigma\}}\,\partial_\mu$ orthogonal to $\{\sigma\}$; $H_{\{\sigma\}}$ is the hamiltonian adapted to $\{\sigma\}$.

The second step is to promote the foliation parametrized by $N_{\{\sigma\}}$, to the status of a dynamical variable in order to replace the frame-dependent dynamical system (12), (14) with a frame-independent one with basic variables ψ_σ, f_σ, $N_{\{\sigma\}}$. For the evolution of f_σ one might tentatively consider the straightforward generalization of (14),

$$\frac{\delta}{\delta\sigma}\,f_\sigma = \frac{\delta}{\delta f_\sigma}\,S_\sigma, \tag{18}$$

but it is not immediately clear how to couple the evolutions of ψ_σ, f_σ and N_σ in a natural way [27].

A different approach to the problem of mantaining relativistic invariance is to abandon the program of associating local beables to the quantum field operators and to turn instead to a search for the "right" observables (§9). One might goes further and ask, as t'Hooft [29], whether there exist a quantum field theory, to be taken seriously as a fundamental theory, which

contains a set of commuting observables invariant under the time evolution, in terms of which all the physical laws can be formulated. These observables would then define the beables of the theory (which by Bell's inequalities would violate local causality [30]).

Such a scheme opens the doors to a way of thinking since long abandoned.

§13. Einstein's approach

We recall Einstein's approach to a fundamental relativistic theory. While it is often presented as in opposition to the quantum one, we consider it as possibly belonging to a deeper level from which a realistic formulation of quantum theory may phenomenologically arise. It is amusing to note that even in the generalized field theory of Einstein, which is certainly a realistic theory, one needs to find beables, the objects which relate the mathematical theory to physical reality.

Einstein's theory [31, 32, 33, 34] is a field theory on the four dimensional continuum based on a minimal set of assumptions. Its phenomenological input is clearly general relativity which is extended by considering as the fundamental field a second order asymmetric tensor g ($g_{\mu\nu} \neq g_{\nu\mu}$) related to a connection Γ having nontrivial torsion ($\Gamma^{\alpha}_{\beta\gamma} \neq \Gamma^{\alpha}_{\gamma\beta}$). Einstein struggled for a long time to obtain field equations which uniquely arise rom the principle of general covariance (Mach's principle [35]).

In a proper gauge ($\Gamma^{\alpha}_{\beta}\alpha = \Gamma^{\alpha}_{\alpha}\beta$) the equations read:

$$\overline{R}(\Gamma) = 0 \quad , \quad d\,\hat{R}(\Gamma) = 0 \ , \tag{19}$$

where $\overline{R}$ and $\hat{R}$ are respectively the symmetric and the antisymmetric part respectively of the Ricci tensor $R \equiv (R_{\mu\nu})$ with the following restriction of the solution manifold of Γ's:

$$g_{\mu\nu,\sigma} - g_{\rho\nu}\,\Gamma^{\rho}_{\mu\sigma} - g_{\mu\rho}\,\Gamma^{\rho}_{\sigma\nu} = 0. \tag{20}$$

One may say that (20), with the introduction of the "potentials" $g_{\mu\nu}$, is the phenomenological part of the theory, but no physical and geometrical interpretations are assumed. In particular it is not a space-time theory, in the sense that no particular pseudo-Riemannian metric is selected a priori. The space-time structure can only be found a posteriori after general solutions have been found. This is, however, a very hard task about which not much is known.

There are claims about the nonphysical character of the theory based on a priori interpretations such as that the symmetric part of g is the metric content of the gravitational field and the antisymmetric part of g is connected with the electromagnetic field [36]. These claims were substantiated by considering test particle motion in the sense of Einstein, Infeld, Hoffman (E.I.H.) [37], which we shall shortly discuss below. It was, for example, been shown that the theory does not describe the motion of electric charges under the Coulomb force [38].

It is now clear that these considerations were to restrictive [39,40]. It is nevertheless worthwhile to understand how a general field theory may yield equations governing the motion of particles. Since the physical situation is clear in general relativity we restrict (19) and (20) to the case of symmetric connections and obtain the Einstein equations in empty space

$$R_{\mu\nu}(\Gamma) = 0 \tag{21}$$

with (20) being solved to yield Γ as Christoffel symbols of the metric g.

Let us recall that in general the Einstein tensor

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\sigma}_{\sigma}$$

has the property of a stress tensor, i.e. its (covariant) divergence is zero: $G_{\mu\nu;\nu} = 0$. One therefore write usually

$$G_{\mu\nu} = \text{const. x } T_{\mu\nu} \quad ,$$

$T_{\mu\nu}$ representing the energy content of matter. Since we have no knowledge about matter, general relativity is not a fundamental theory, and Einstein hoped to arrive at a fundamental one with the generalized field theory based on (19) and (20). He certainly assumed that quantum physics was contained in his theory. A convincing argument for him might have been the strength of nonlinear equations to predict the motion of singularities, i.e. matter points.

§14. Test particle motion

Outside of matter (21) gives ten equations for ten $g_{\mu\nu}$'s; four components of g are to be free due to the free choice of the coordinate system (gauge invariance), but this does not lead to

inconsistencies because of Bianchi identities. One may then consider the solution manifold of the problem where (21) holds outside a finite number of world tubes around the curves $q_1(s), \ldots, q_N(s)$ and ask whether for the physical solution (positive stress energy tensor) the tubes are determined by (21) i.e., whether the theory predicts the motion of "massive points". This question occupied Einstein for a long time; a good source for the analysis is [41].

One considers a slow motion approximation,

$$\frac{d\, q_n^i(s)}{ds} \sim \lambda \,, \qquad \lambda \downarrow 0 \,, \qquad i = 1, 2, 3 \,, n=1, \ldots N$$

combined with a weak field analysis,

$$g(\lambda) \doteq \sum_{k=0}^{\infty} \lambda^k \, g_k(m_1, \ldots, m_N; q_1(s), \ldots q_N(s)), \tag{22}$$

where the perturbative solution depend on parameters $m_1, \ldots, m_N$ (integration constants) and the $q_n(s)$'s. Given a soution (22) of (21) one requires the solution to be "physical", namely to correspond in the nonrelativistic limit to a Newtonian solution. Then taking $m_n > 0$, $n = 1, \ldots, N$, one finds the desired constraints among $q_1(s), \ldots, q_N(s)$ and obtains in fact the newtonian force, with relativistic corrections, between the masses m_n's moving along the trajectories $q_n(s)$'s.

It is worthwhile to remark that the above analysis may be applied to other equations as well. It certainly would be desirable to obtain the E.I.H. result in the modern language of scaling limits, as it is the case for test particle motion in classical mechanics [42]. To our knowledge not much rigorous work has been done in this direction.

§15. Concluding remarks

The points we wish to emphasize are:

Bohm's theory is a complete theory, it eliminates the artificial distinction between apparatus and quantum system of the orthodox interpretation (the "speakable" and the "unspeakable" of Bell). The building blocks of the theory are "beables"; in the non relativistic case: real waves and real particles having real positions in space. The theory gives, in the non-relativistic case, a complete occount of nonrelativistic quantum physics (spin and external electromagnetic fields may also be easily included [6, 16]): Its predictions agree with the quantum-mechanical ones and are solely based on the properties of the dynamical system (2), (3). Probability enters, as in classical mechanics, only through ignorance of the initial conditions. The capability of (2), (3) in describing a large variety of physical situations, including measurements, and thus explaining the role played by self-adjoint operators in orthodox quantum mechanics, should perhaps be compared with the explanatory power of the classical dynamical system

$$\frac{dp_n}{dt} = -\frac{\partial}{\partial q_n} H(p_1, \ldots, p_N; q_1, \ldots, q_N) \tag{22}$$

$$\frac{dq_n}{dt} = \frac{\partial}{\partial p_n} H(p_1, \ldots, p_N; q_1, \ldots, q_N) \tag{23}$$

which occounts equally well for a large class of physical situations , e.g. gravitation and thermodynamics. (There is more than an analogy here since the study of the relation between (2), (3) and (22), (23) is the proper setting fo the study of the classical limit [43]).

Finally Bohm's theory has the merit of opening itself to further questions regarding a more fundamental relativistic theory, for example one based only on geometrical notions. On a higher level the real waves, the real particles and the nonlocal character of nature could find their proper setting in Einstein's theory. It is at the moment mere speculation of how this could come about: perhaps the notion of wave function plays a role in the analysis of the solution manifold of Einstein equations by parametrizing it in a similar way as the masses do in the E.I.H.'s analysis.

What we spell out here has, in one form or another, been subject to recent considerations: Nelson puts forward the "background field hypothesis": quantum fluctuations are the result of a classical field interaction [14]. (The first of a list of open problems in [14] is: "To find a classical lagrangian of system + background field oscillators + interaction, that with reasonable initial probability measures and in the limit as the cut-offs on the background field are removed, produces a conservative diffusion in the system (or to show that this is impossible ...)" — By "conservative diffusion" it is essentially meant the dynamical system (2), (3) with $\vec{V}_n$ given by (9) and satisfying (10) —). t'Hooft suggests an equivalence between quantum mechanical systems and classical deterministic systems; in order to substantiate the idea he analyzes simple models which might exhibit such a property [29]. Galgani and coworkers, inspired by ideas of Nernst and Jeans, appeal to recent K.A.M.-related results suggesting that the lack of ergodicity of classical models describing the interaction of matter and radiation could be responsible of the quantum thermodynamical behaviour, as the one described by Planck distribution [44]. (Since no satisfactory classical relativistic theory of matter and radiation exists, one considers the standard Maxwell-Lorentz theory with cut-offs which break relativistic invariance. Thus the ultimate theory to study here is a unified theory in the sense of Einstein.) Penrose, accepting the truth of nonlocality aims at a geometrical theory, not necessarly based on the four dimensional continuum (twistor theory and/or spin-networks theory), which embodies nonlocality and goes beyond quantum mechanics and general relativity [46].

§16. References

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