Abstract

1. Ramajujan's tau function is given by $\tau(n) = A(n) - B(n)$, where $A(n) := \frac{65}{756}\sigma_{11}(n) + \frac{691}{756}\sigma_5(n); B(n) := 691\sum_{j=1}^{n-1}\sigma_5(j)\sigma_(n-j)$. It suffices therefore to prove $\tau(n) \neq 0$ for prime p, since $\tau(n)$ is multiplicative.

2. $A(p)$ has a prime factor $q > p$.

3. We construct a matrix $[a_{i,k}]$ modulo q. Then $\tau(p) \neq 0$ is equivalent to the additive group $\{\sum_{k=0}^{q-1} ka_{i,k}\}_{i=0}^{q-1}$ forms a nontrivial group.

4. We show that $\{\sum_{k=0}^{q-1} ka_{i,k}\}_{i=0}^{q-1}$ indeed forms an additive group of order q.

5. From #4 above, Lehmer's Conjecture follows.