Games For Arbitrarily Fat Rats

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ABSTRACT.

In kindergarten we learned about the integers (Peano axioms); in grammar schhol – about pairs of integers (rationals); and then in high school, about the reals (Dedekind cuts). Berlekamp, Conway, Guy discovered and promoted a method (Don Knuth: "Surreal Numbers") of creating all of those and much more – namely games! – in one masterful stroke.

Yet the rationals sometimes present obstinate difficulties often overlooked. Example. Let $1 < \alpha_1 <, \ldots, < \alpha_m$ be real numbers, dubbed *moduli*, $m \geq 3$. An over 40 years old conjecture states that there exist reals γ_i such that the system $(\lfloor n\alpha_1 + \gamma_1 \rfloor, \ldots, \lfloor n\alpha_m \rfloor + \gamma_m \rfloor)$ constitutes a complementary system of m sequences of integers if and only if $\alpha_i = \lfloor (2^m - 1)/2^{m-i} + \gamma_i \rfloor$, $i = 1, \ldots, m$. It is known that for integers and irrationls, 2 moduli have to be equal, but the problem is wide open for the rationals.

We have created, for every $m \geq 2$, an *invariant* game whose P-positions (2nd player win positions) are the conjectured moduli, and gave game rules and an efficient strategy for the next winning move if not in a P-position. Motivation: (1) "Play" with the above conjecture. (2) Find efficient game rules for games defined only by their sets of P-positions. (Rats: rationals.)

Joint with Urban Larsson.