Combinatorial interpretations of binomial coefficient analogues related to Lucas sequences

by

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Let s and t be variables. Define polynomials $\{n\}$ in s, t by $\{0\} = 0$, $\{1\} = 1$, and $\{n\} = s\,\{n-1\} + t\,\{n-2\}$ for $n \geq 2$. If s, t are integers then the corresponding sequence of integers is called a *Lucas sequence*. Define an analogue of the binomial coefficients by

$$\left\{ {n \atop k} \right\} = \frac{\{n\}!}{\{k\}!\,\{n-k\}!}$$

where $\{n\}! = \{1\}\,\{2\}\cdots\{n\}$. It is easy to see that $\left\{ {n \atop k} \right\}$ is a polynomial in s and t. We give two combinatorial interpretations for this polynomial in terms of statistics on integer partitions inside a $k \times (n-k)$ rectangle. When $s = t = 1$ we obtain combinatorial interpretations of the fibonomial coefficients which are simpler than any that have previously appeared in the literature.