Game Theory – An Alternative Mathematical Experience

Ein-Ya Gura

The Federman Center for the Study of Mathematics

The Hebrew University of Jerusalem

Few branches of mathematics have been more influential in the social sciences than game theory. In recent years, it has became an essential tool for all social scientists studying the strategic behavior of competing individuals, firms, and countries.

However, the mathematical complexity of game theory is often very intimidating for students who have only a basic understanding of mathematics.

Here is offered a course in game theory which addresses this problem by providing students with an understanding of the key concepts and ideas of game theory without using formal mathematical notation.

The course investigates four areas of game theory by using four different topics;

1. College admissions.
2. Social justice and majority voting.
3. Coalitions and cooperative games.
4. A bankruptcy problem from the Talmud.

In the 21^{st} century game theory stands in the front of the interdisciplinary research and yet the high-school's mathematics curriculum ignores completely this branch of mathematics.

Game theory undertakes to build mathematical models and draw conclusions from these models in connection with interactive decision-making; situations in which a group of people not necessarily sharing the same interests are required to make a decision.

Mathematical modeling is the essence of the teaching of mathematics. The problem is that most of the mathematics taught at school has no connection to real-life situations and therefore the mathematical modeling is quite meaningless.

Game theory is a branch of mathematics that is motivated mostly by the social sciences or, better by human behavior and therefore constructing mathematical models for real-life situations is natural to the theory.

In game theory one can ensure that the students are able to understand the basic problem on which they are working.

Our main goal was to make mathematics a subject that can be discussed and thought about through a basic comprehension of the problem at hand.

The Course

The course follows the book *Insights into Game Theory: An Alternative Mathematical Experience* written by late Michael Maschler and Ein-Ya Gura, published by Cambridge University Press in 2008.

The choice of topics reflects our purpose: we wanted to present material that does not require mathematical prerequisites and yet involves deep game-theoretic ideas and some mathematical sophistication.

Broadly speaking, the topics are all related to the various meanings that can be given to the concept of “fair division”.

The course is a collection of few topics from the theory that are intended to open a window onto new and fascinating world of mathematical applications to the social sciences.

It selects a small number of topics and studies them in depth. It shows the student how mathematical model can be constructed for real-life issues.

One of the aims of the course is to acquaint the student with “a different mathematics”, a mathematics that is not buried under complicated formulas, yet contains deep mathematical thinking.

Another aim is to show that mathematics can efficiently handle social issues.

A third aim is to deepen the mathematical thinking of the person who studies this book. We believe that by studying the topics of the book, the mathematical thinking of the student will be enriched.

The book on which the course is based is divided into chapters that are independent so that a teacher and a student can choose one chapter or several and cover them in any order.

The first chapter, “Mathematical Matching” concerns, among other things, the problem of assigning applicants to institutions of higher learning. Each applicant ranks the universities in which he is applying according to his scale of preferences. The universities, in turn, rank the applicants for admission according to their own scale of preferences. The question is how to effect the “matching” between the applicants and the universities. The problem leads to unexpected solution.

The second chapter, “Social Justice”, concern social decision rules. In a democratic society it is customary to make decisions by a vote. The decision supported by the majority of voters is adopted. But we show that “majority rule” does not always yield a clear-cut solution. The attempt to find other voting rules raises un expected difficulties.

The third chapter, “The Shapley Value in Cooperative Games”, addresses, among other things, the following problem: a group of people come before an arbitrator and inform him of the expected profits of every subgroup, as well of the whole group, if the groups operate independently. It seems that these data are sufficient for the arbitrator to decide how to divide the profits if all litigants operate together.

The fourth chapter, “Analysis of a Bankruptcy Problem from the Talmud”, addresses the following problem: several creditors have claims to an estate, but the total amount of the claims exceeds the value of the estate. How should the estate be divided among the creditors? In the chapter several solutions are accepted, two of which are discussed in the Talmud.

Broadly speaking all the chapters in the book represent attempts at reaching a decision in a conflict situation and in each of them we show the difficulties in trying to define a “superior” solution.

The first chapter on matching presents a “weak” condition of stability, which nevertheless yields many matchings. One of them is best for the men and another is best for the women.

The second chapter tries to reach a decision by voting and we saw that the fair voting rule is not always possible.

The third chapter has probably the most successful solution. It provides a solution for an unbiased arbitrator, by supplying axioms that seem fair. However, somewhat different axioms, not covered in this book, yield different solution.

Finally, the fourth chapter, which consider a case of bankruptcy conflicts, shows that even in this simple case a superior solution cannot be found.

In conclusion, we see that various solutions are well tailored to many real situations, but there is no single solution that fits all situations. Each solution sheds some light on reality.