MATH 252 - Readiness Assessment 4

June 15, 2018

Name: _____

1. Let $x(t)$ and $y(t)$ be two functions defined as the solution to a two-component autonomous system

$$\frac{dx}{dt} = f(x, y) \qquad \frac{dy}{dt} = g(x, y)$$

What does this system look like if it is linear with constant coefficients?

2. What is/are the equilibrium solutions of these systems? There are two different options here, depending on some condition on the coefficient matrix A.

3. Are the vectors $\begin{pmatrix}1\\2\end{pmatrix}$ and $\begin{pmatrix}2\\-1\end{pmatrix}$ linearly independent?

4. Are the solutions $\begin{pmatrix}2e^t - e^{-2t}\\ e^t + e^{-2t}\end{pmatrix}$ and $\begin{pmatrix}e^t + e^{-2t}\\ -e^{-2t}\end{pmatrix}$ linearly independent?

5. Which of the following solutions for $x(t), y(t)$ travel on a straight line through the origin?

$$\begin{pmatrix}1\\3\end{pmatrix} e^{2t} \qquad \begin{pmatrix}1\\2\end{pmatrix} e^t + \begin{pmatrix}2\\3\end{pmatrix} e^{-4t} \qquad \begin{pmatrix}1\\2\end{pmatrix} e^{3t} + \begin{pmatrix}2\\4\end{pmatrix} e^{-t}$$

$$\begin{pmatrix}-1\cos(t)\\ 2\cos(t)+\sin(t)\end{pmatrix} + \begin{pmatrix}2\sin(t)\\ 4\cos(t)-3\sin(t)\end{pmatrix} \qquad \begin{pmatrix}1\\-1\end{pmatrix} e^{-t}$$

6. If $e^{\lambda t}\vec{v}$ is a straight line solution to $\frac{d\vec{x}}{dt} = A\vec{x}$, what must λ and $\vec{v}$ satisfy? What are λ and $\vec{v}$?