MATH 252 - Problem Set 8

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Part 1

For each of the following matrices, use the Trace-Determinant Plane to determine what kind of equilibrium solution the origin is if this matrix was used as the coefficient matrix in

$$\frac{d\vec{x}}{dt} = A\vec{x}$$

After you get through them, find a few general solutions and draw phase portraits to confirm your answer.

1. $\begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$ **2.** $\begin{bmatrix} -3 & 2 \\ 1 & -3 \end{bmatrix}$ **3.** $\begin{bmatrix} 4 & 0 \\ -3 & -1 \end{bmatrix}$ **4.** $\begin{bmatrix} 0 & -2 \\ 7 & 1 \end{bmatrix}$ **5.** $\begin{bmatrix} 3 & -1 \\ -2 & -1 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$ **7.** $\begin{bmatrix} -2 & 4 \\ 9 & -1 \end{bmatrix}$ **8.** $\begin{bmatrix} 1 & 5 \\ -2 & -1 \end{bmatrix}$ **9.** $\begin{bmatrix} 2 & -5 \\ 1 & -1 \end{bmatrix}$ **10.** $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

Part 2

In this section, you will analyze one-parameter families of systems in groups. Each group will take one of the following problems, work out the details, and then present it to the class. The components of the answer that I want to see are as follows:

1. Calculation of the trace and determinant of the family.
2. A sketch of the curve traced out by this one-parameter system in the Trace-Determinant Plane and how you determined this curve.
3. An identification of where the type of critical point at the origin changes, as well as what it changes from and to at that point.
4. (Optional) A few phase portraits of the system at specific values of the parameter μ.

The problems are:

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 3-2\mu \\ -1 & \mu \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 0 & -1 \\ e^{\mu} & \mu \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \mu & \mu \\ \mu+6 & 2 \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \mu & -1 \\ 2 & 1 \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} \mu & -1 \\ 2 & -1 \end{bmatrix}$$

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 3-\mu^2 & \mu+1 \\ 2-\mu & 1 \end{bmatrix}$$