Triangles and Spheres - An Introduction to Curvature

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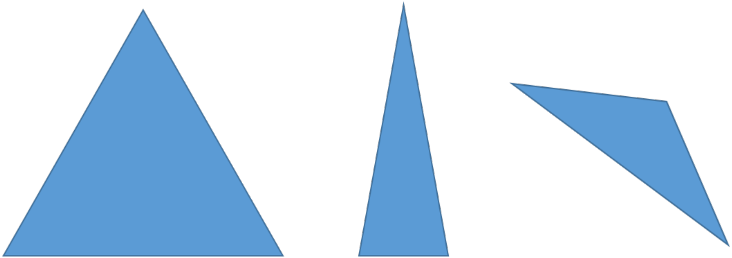
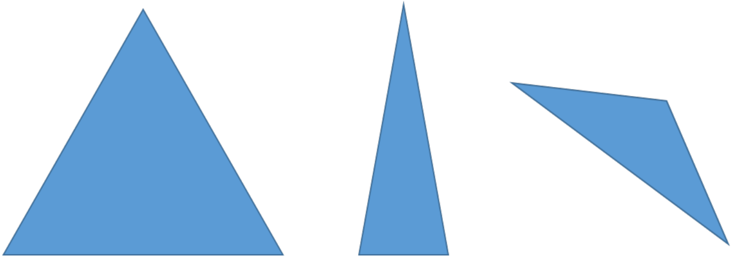
Triangles

Today we're going to be talking about geometry, in particular the geometry of triangles. You may think that triangles are simple, but they can actually tell us a lot about geometry, including some things you probably wouldn't actually learn about until college. So this is a sneak peek into some pretty cool math stuff.



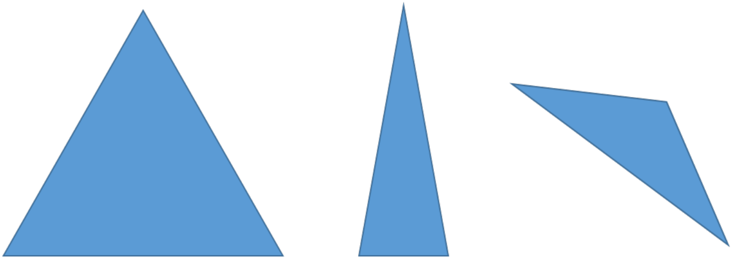
Triangles

So, here's how we're going to start. All of you should have some paper and a protractor. Draw a few triangles on the paper of a variety of sizes and shapes, Remember, triangles have straight sides. Use the protractor to measure all of the angles, and see what they add up to for each of the triangles you drew. Compare with the people around you.

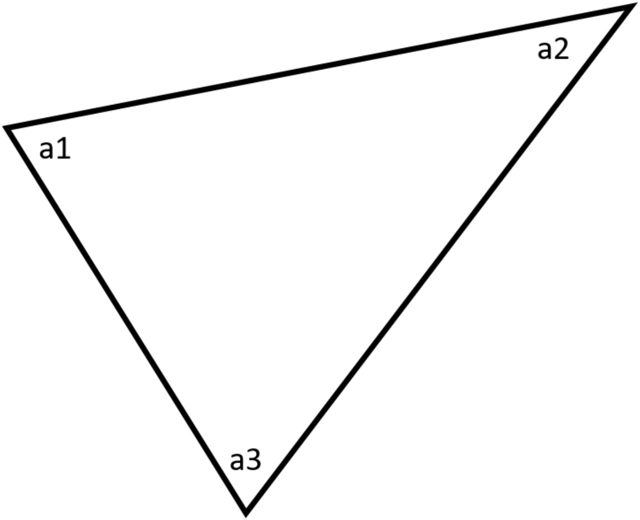


Surprise - I know what you got!

So, what you probably discovered is that everyone got something around 180 for the total sum of the angles. It didn't matter what type of triangle you drew, it was always around 180, and the error there is just approximation/measurement error.



Proof of 180 Angle Sum



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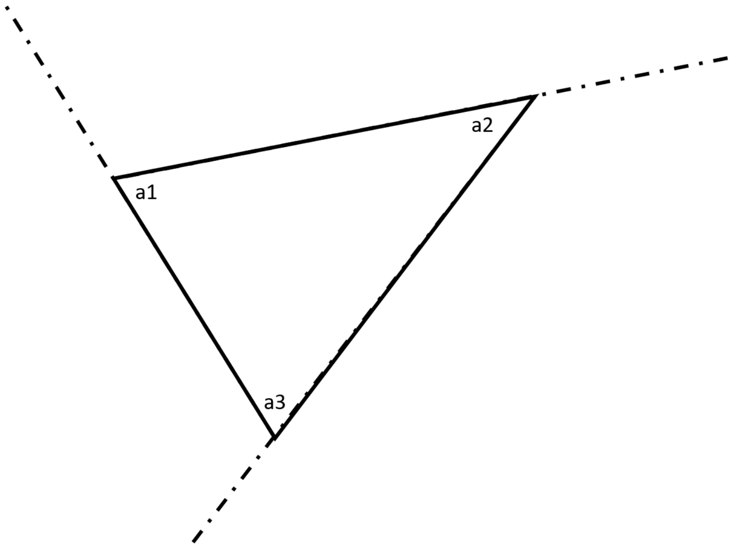

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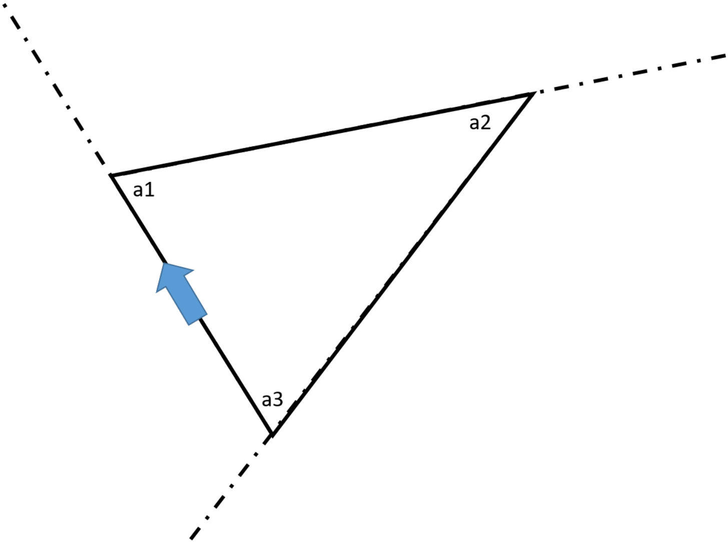

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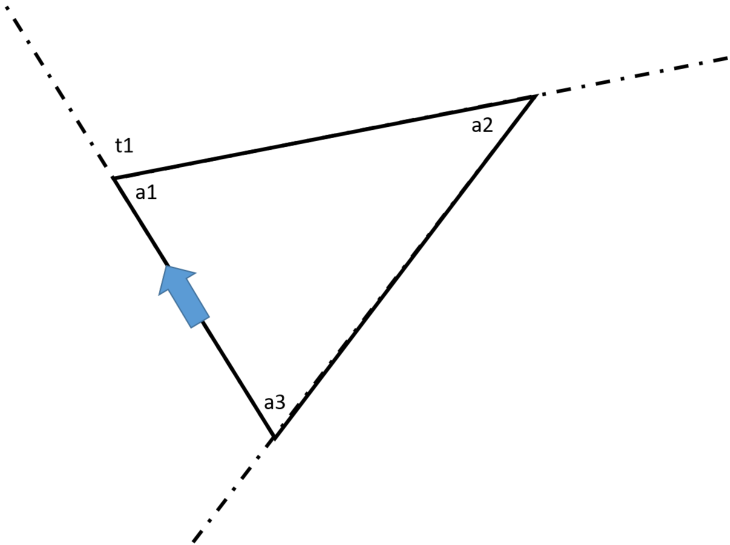

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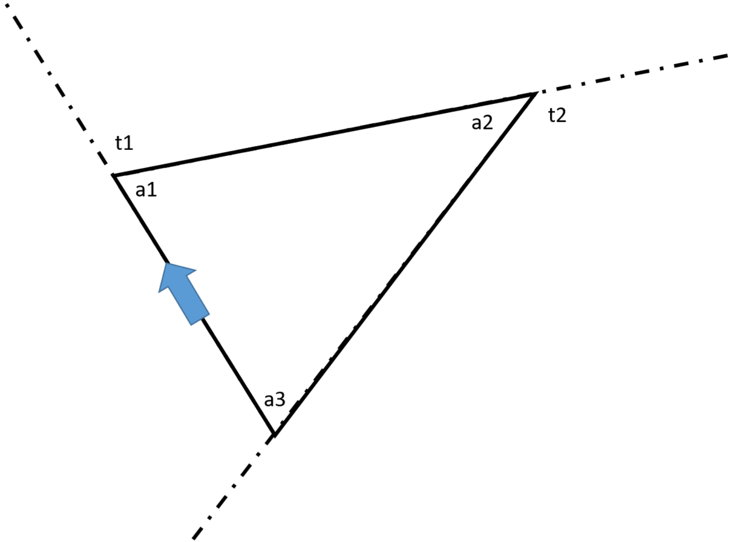

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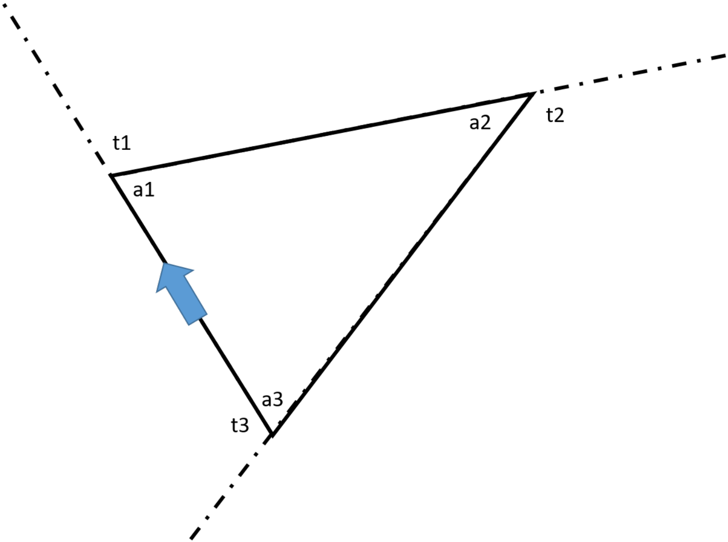

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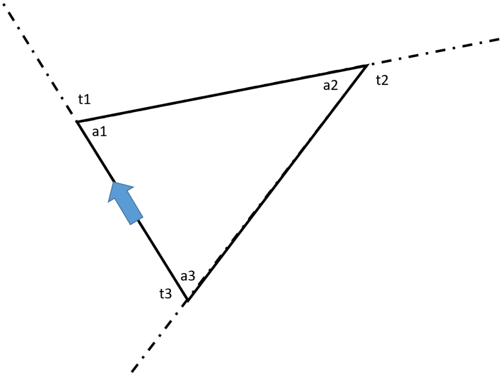

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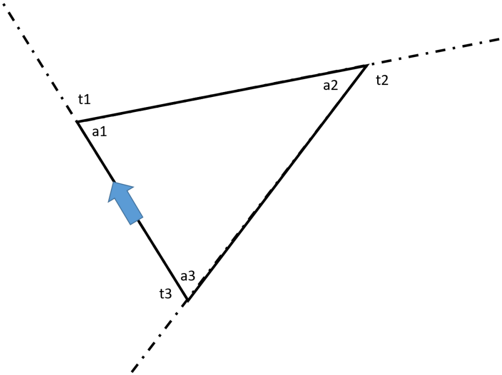

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$$a_1 + t_1 = 180$$
$$a_2 + t_2 = 180$$
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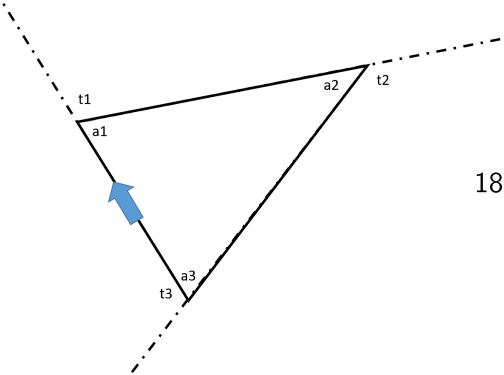

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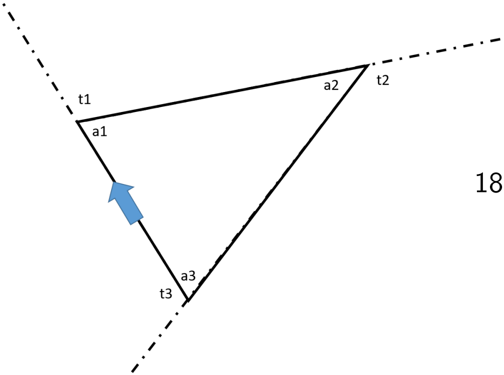

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It turns out you can get basically anything from 0 to 540. Draw some more triangles and see how small or large you can get the angle sum to be.

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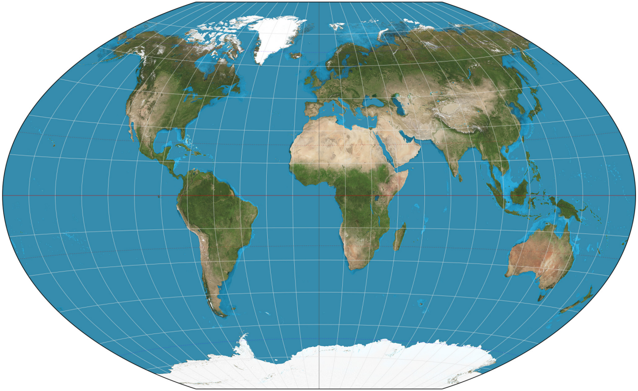
A curve is a *geodesic* if it minimizes the distance between points. So, we have a statement like "The geodesics of a flat piece of paper are straight lines."

Geometry of a Sphere

Now, instead of a piece of paper, we want to do the same on a sphere. Based on what we just talked about, knowing what the "straight lines" are on a sphere is important. This is not nearly as obvious as it was in the plane. Mainly because if I draw a straight line on a sphere, and then rotate the sphere, it won't look straight anymore.



Minimizing Distance on a Sphere and Triangles

Now, for the next part of this activity, you're going to get to figure out what the "straight lines" look like on a sphere, as well as work with some triangles. There are two parts to this:

1. Everyone will get a small beach ball. On that, you should mark a few points and try to figure out the *geodesic* between them. You have a piece of string that you can use to figure out what path will minimize distance.
2. Each group will have a large beach ball that I have drawn some "triangles" on. Measure the angles, find the angle sum, and pass it around to the other members of your groups

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- Triangles always have angle sum bigger than 180.
- The amount the angle sum exceeds 180 is proportional to the area of the triangle.

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- Positive curvature - the angle sum increases as area increases.
- Zero curvature - the angle sum does not change as area increases.
- Negative curvature - the angle sum decreases as area increases.

Thanks for your attention!

The mini-beach balls are for you to keep.
I want the big ones back.