

# MATH 252 - Bifurcation Jigsaw

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## Purpose

The purpose of this activity is to allow you, as groups, to explore bifurcations and the different types that exist by analyzing a problem on your own. It will also give you an opportunity to improve your skills at talking about math concepts with your peers, as you will be presenting your discoveries to the other students in the class.

## Structure

This activity will have three parts. During the first part, you and your group will work on analyzing two different one-parameter families of ODEs to search for bifurcation points. Each group will be working on the same problems here, and after some time to figure it out, we will discuss the results, so you know what the answer to a problem like this looks like. During the second part of this activity, each group will analyze another bifurcation problem, but each group will have a different problem. As a group, you will be responsible for finding and analyzing the bifurcation points of your problem following the format below. Once I see that everyone has the proper results, we will move on to the last part of the activity. Finally, in the 'Jigsaw' part of the activity, the groups will be shuffled so that every new group has at least one person in it who did each of the problems. Then, everyone in the new group will take a turn sharing what they discovered about the problem they analyzed. That way, at the end of class, everyone will have heard about all of the problems.

## Format of Answer

Given a one-parameter family of ODEs, your analysis of any bifurcation points should contain the following.

1. A determination of any/all bifurcation points of the family of equations. This may include sketches of the graph of the function  $f_\mu(y)$ .
2. For each of the bifurcation points, a phase line (with solution sketches) for a value slightly larger and slightly smaller than the bifurcation point.
3. A description of what happens at the bifurcation points. (How many equilibrium points are there before and after? What type are they?)
4. A sketch of the bifurcation diagram for this family.

In terms of turning in this assignment, each group will need to hand in one copy of their bifurcation analysis for the second part of this activity. It can be handed in today or at the start of class on Friday.

## Problems

For the initial part of this activity, each group will analyze the families

$$\frac{dy}{dt} = y^2 - \mu y$$

and

$$\frac{dy}{dt} = \mu - y^3$$

For the second phase, the possible families of ODEs are

1.  $\frac{dy}{dt} = \mu y - y^3$
2.  $\frac{dy}{dt} = 2y \left(1 - \frac{y}{10}\right) - \mu$
3.  $\frac{dy}{dt} = y^2 - \mu y + 1$
4.  $\frac{dy}{dt} = (y^2 - \mu)(y^2 - 9)$
5.  $\frac{dy}{dt} = y^3 + \mu y^2$
6.  $\frac{dy}{dt} = y^4 + \mu y^2$

where, in each case,  $\mu$  is the parameter to be varied.